统计物理学在金融动力学中的应用

-- 多体微观模型

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Financial dynamics

1. J.J. Chen and B. Zheng, Agent-based model with asymmetric trading and herding in complex financial dynamics, submitted to Plos One, 2013
2. F. Ren, B. Zheng, and P. Chen, Modeling interactions of trading volumes in financial dynamics, Physica A389 (2010) 2744
3. F. Ren, B. Zheng, T. Qiu and S. Trimper, Minority games with score-dependent and agent-dependent payoffs, Phys. Rev. E74 (2006) 041111
4. F. Ren, B. Zheng, T. Qiu and S. Trimper, Score-dependent payoffs and Minority Games, Physica A371 (2006) 649
5. L.X. Zhong, D.F. Zheng, B. Zheng and P.M. Hui, Effects of contrarians in the minority game, Phys. Rev. E72 (2005) 026134.
6. B. Zheng, F. Ren, S. Trimper and D.F. Zheng, A generalized dynamic herding model with feed-back interactions, Physica A343 (2004) 653
7. B. Zheng, T. Qiu and F. Ren, Two-phase phenomena, minority games, and herding models, Phys. Rev. E69 (2004) 046115

Publications

9. 郑波，金融动力学的时空关联与大波动特性
-- 兼谈中西方金融市场的对比研究，
《物理》第39卷（2010年）第95页.
10. 郑波，金融市场的微观动力学及其数值模拟研究，
《管理学报》第6卷（2009年）第1608页

Contents

微观多体模型

* 羊群模型

股民结群，相互作用从简单到复杂，五花八门

* 多体博弈论

多体不完全信息博弈，思路清晰，但是离真实市场有一定距离

* 订单模型

出发点比较实际，但需要附加假设

* Ising类模型
* 随机过程模型

金融微观模型的现状

* 可以取得局部成功，但整体不令人满意。波动率的运动规律相对简单，而价格本身的运动规律极其艰难

* 微观多体模型加上经验参数较令人信服。例如**Stanley**等用市场数据确定模型参数

PNAS 109(2012)8388

Herd behavior in a complex adaptive system

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Contributed by H. Eugene Stanley, April 7, 2011 (sent for review September 13, 2010)

In order to survive, self-serving agents in various kinds of complex adaptive systems (CASs) must compete against others for sharing limited resources with biased or unbiased distribution by conducting strategic behaviors. This competition can globally result in the balance of resource allocation. As a result, most of the agents and species can survive well. However, it is a common belief that the formation of a herd in a CAS will cause excess volatility, which can ruin the balance of resource allocation in the CAS. Here this belief is challenged with the results obtained from a modeled resource-allocation system. Based on this system, we designed and conducted a series of computer-aided human experiments including herd behavior. We also performed agent-based simulations and theoretical analyses, in order to confirm the experimental observations and reveal the underlying mechanism. We report that, as long as the ratio of the two resources for allocation is biased enough, the formation of a typically sized herd can help the system to reach the balanced state. This resource ratio also serves as the critical point for a class of phase transition identified herein, which can be used to discover the role change of herd behavior, from a ruinous one to a helpful one. This work is also of value to some fields, ranging from management and social science, to ecology and evolution, and to physics.

experimental econophysics | computational econophysics | market-directed resource-allocation game | minority game | agent-based model

allocation system. Accordingly, herd behavior is commonly seen as a tailor-made cause for explaining bubbles and crashes in a CAS with the existence of extremely high volatility. But is this "common sense" always right? Based on results of this study, we argue that herd behavior should not be labeled like the killer of balance and stability all the time. Here we focus on the effect of herding on the whole CAS for resource allocation, because it is most important for as many agents (involving human beings) as possible to survive in various kinds of CASs like social, ecological or biological systems. Therefore, we shall not study or consider the details on how to reach a herd through contagion and/or imitating. In fact, our results are not dependent on the process of herding formation.

Experiment

We design and conduct a series of computer-aided human experiments, on the basis of the resource-allocation system (4, 11–13), in order to study the necessary conditions for a CAS to reach the ideal balanced state. Using this kind of experimental settings will allow us to investigate the herd behavior in a well regulated abstract system for resource allocation, which reflects the fundamental characteristics of many CASs (14–17). Human participants of the resource-allocation experiment are students recruited from several departments of Fudan University. Before the start of experiments, a leaflet (as shown in *SI Text: Part I*) was provided which explains configurations of the experiment and

应用微观模型解释宏观和介观模型, e.g.,

* Relations between price returns, trading volume and number of trades.

* Leverage and Anti-leverage effects

Price return, trading volume and number of trades

Gabaix, Gopikrishnan, Plerou and Stanley, Nature 423, 267 (2003)

$$p[|\Delta Y(t)| > x] \sim x^{\zeta_r}$$

$$p[V(t) > x] \sim x^{\zeta_V}$$

$$p[N(t) > x] \sim x^{\zeta_N}$$

$\zeta_{\mathrm{r}} \approx 3$ $\zeta_V = 1.5$ $\zeta_N = 3.4$ $\zeta_r = 2\zeta_V \approx \zeta_N$

i.e., $|\Delta \mathrm{Y(t)}| \sim \mathrm{N(t)} \sim \sqrt{\mathrm{V(t)}}$

A phenomenological model

* large market participants (mutual funds) control the financial markets

  initiate buying with large volume
  induce price increasing

* perform in an optimal way

  sell in a time with maximum benefit

Contents

General formulation of microscopic modeling

$V_i(t)$: **trade volume of agent *i***

$\sum_i \pm V_i(t)$: **volume imbalance,**
it relates to the price return

Dynamics

$$V_i(t+1) = F_i[\{V_j(t)\},\ \ h_i(t)]$$

e.g.,

$$V_i(t+1) = K_{ij}V_j(t) + h_i(t)V_i(t)$$

In simple words, at time t,

which $V_i(t) = 0$

which $V_i(t) > 0$

which $V_i(t) < 0$

Minority Game

History : m **time steps,** 2^m **states**

Strategies: 2^{2^m}

N_a **agents** N_p **producers**

S strategies **1 strategy**

and **inactive**

Price change: $\triangle Y(t')$ **= buyers - sellers**

Scoring **:** **minority wins** $|\triangle Y(t')|$ **points**

Forget the trade volume and trading times first

This Minority game explains some stylized fact of financial markets including long-range correlation

But suffers from periodic character

Development

- History in detail
- Changing strategies
- Minority vs majority games
- Scoring for strategies
- Stochastic interactions

……

Our work

Score- and agent-dependent payoffs for strategies

Ren, Zheng, Qiu and Trimper, **Phys. Rev. E74** (2006), 041111

Payoffs both score- and agent-dependent

-- scoring for strategies

$$U_{i,s}(t+1) = U_{i,s}(t) + g_{i,s}(t)$$

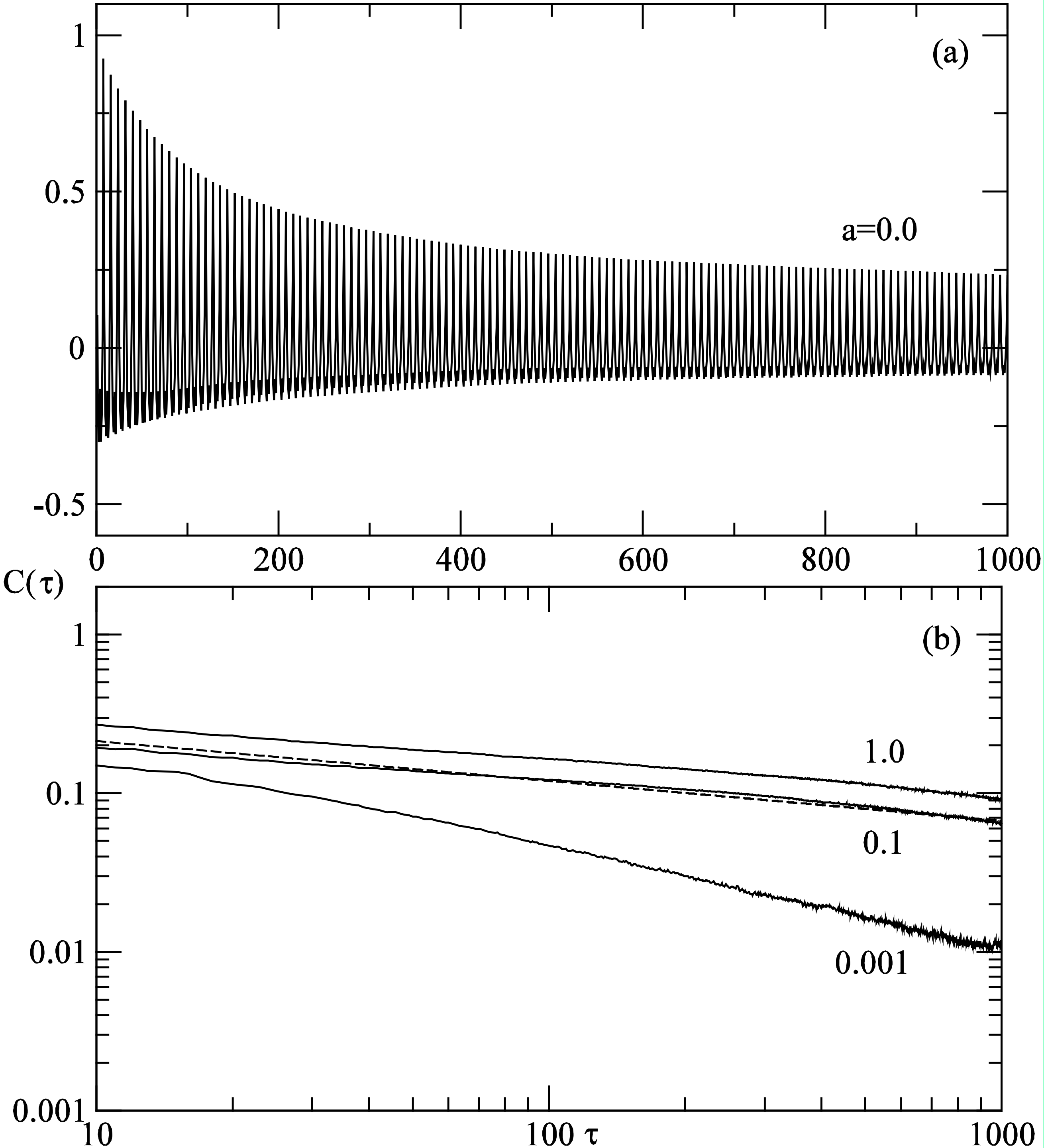
$$g_{i,s}(t) = -\sigma_{i,s}^{\mu(t)} A(t)\; b \frac{\exp(aU_{i,s}(t))}{\sum_{s=1}^{S} \exp(aU_{i,s}(t))}$$

Price change

$A(t)$=buyers-sellers

Agent-dependence of $g_{i,s}(t)$ is essential!

It suppressed the periodic feature, better than the inactive strategy.


(a)
a=0.0
1
0.5
0
-0.5
0
200
400
600
800
1000
C(τ)
(b)
1
0.1
0.01
0.001
1.0
0.1
0.001
10
100 τ
1000

Contents

EZ model : **Eguiluz and Zimmermann, Phys. Rev. Lett. 85 (2000)5659**

N **agents, at time *t*, pick agent *i***

1) **with probability *1-a*, connect to agent *j*, form a larger cluster;**

2) **with probability *a* , cluster *i* buy (sell), resolve the cluster *i***

Magnitude of price return :

$$|\triangle Y(t')| = \text{size of cluster } i$$

This herding model explains the power-law decay (fat-tail) of *P(Z, t)*, but NOT the long-range correlation

Interacting herding model

Zheng, Qiu and Ren, **Phys. Rev. E69** (2004), 046115

1/a **: rate of information transmission**

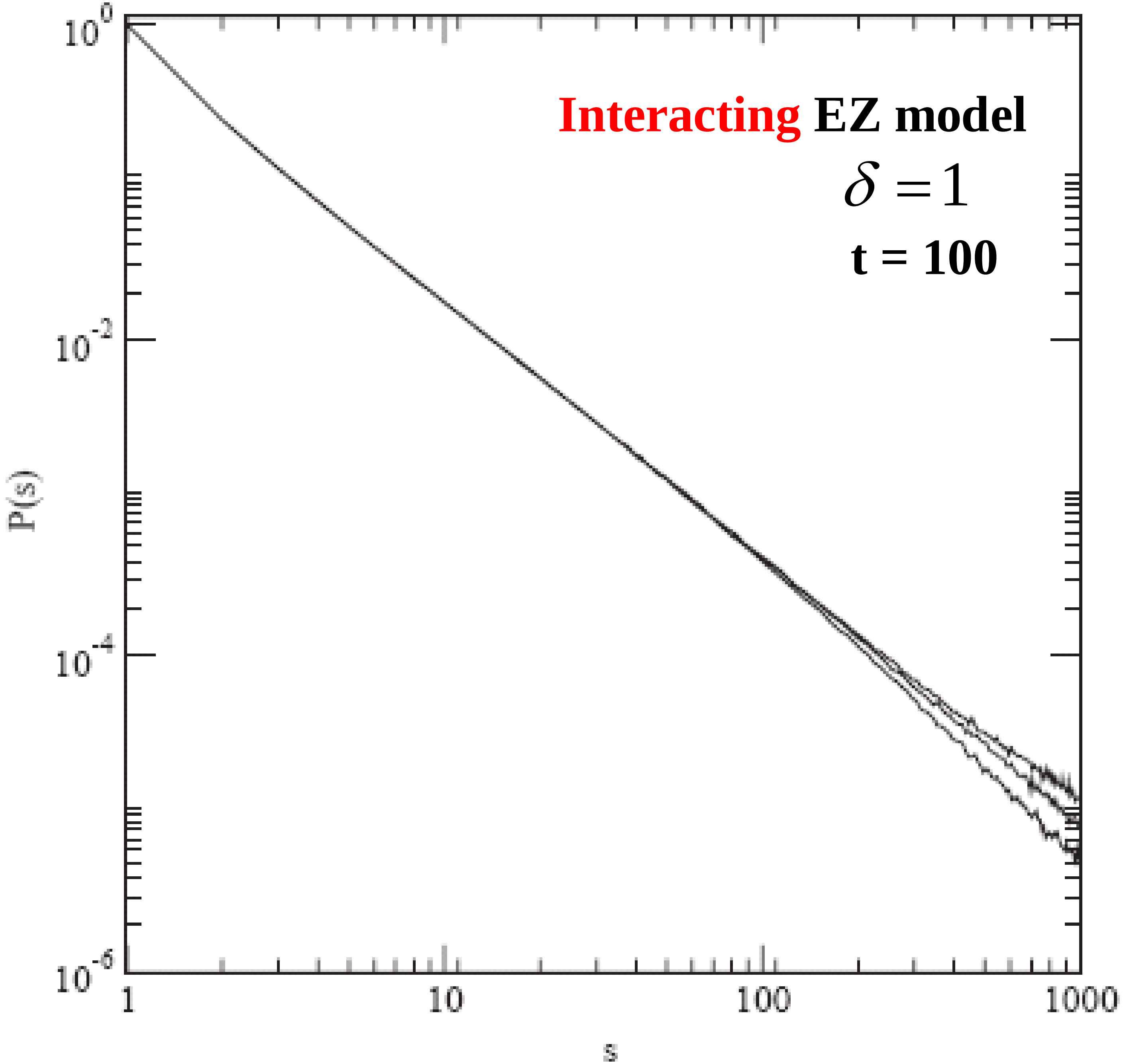
Dynamic interaction

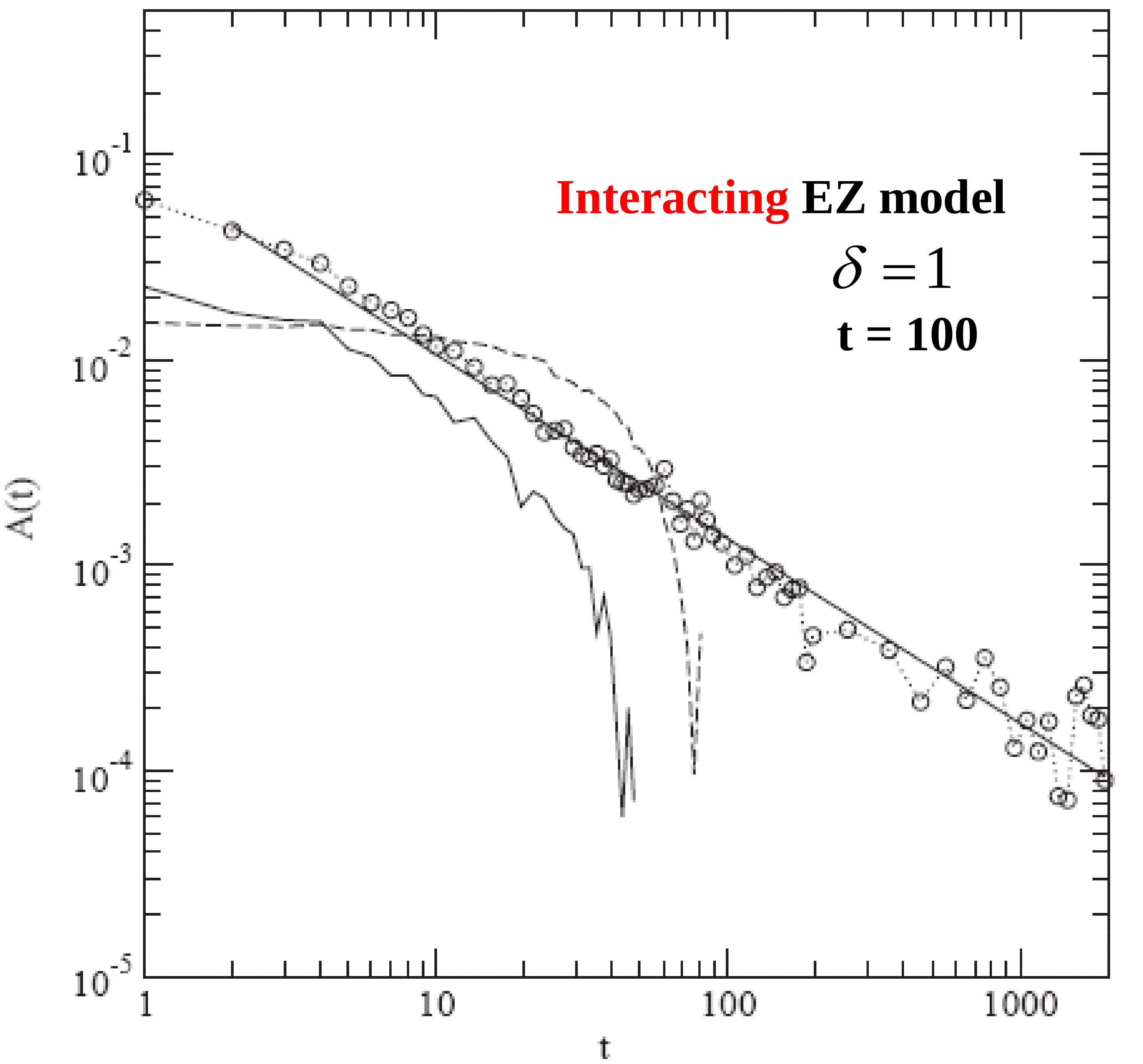
$$a = b + c / s$$

1/b **is the highest rate**

- *** take a small *b***
- *** fix *c* to the 'critical' value :**

  P(Z,t) **obeys a power-law**


Interacting EZ model
$\delta = 1$
t = 100
P(s)
s
10^0
10^{-2}
10^{-4}
10^{-6}
1
10
100
1000



Interacting EZ model
$\delta = 1$
t = 100
A(t)
t
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
100
1000

Introduce trading volume

Ren, B. Zheng, Chen, **Physica A389 (2010) 2744**

$$a_i(t) = 1/[1 + bV(t-1)/v_i(t-1)]$$

$$v_i(t) = 1/a_i(t)$$

One then obtains trading volume
& number of trades

Introduce price return

give + or – sign to each cluster

at time *t* , pick agent *i*

1) **with probability** $1-a_i$ **, connect to agent *j*,**
   form a larger cluster, with majority rule

2) **with probability** a_i **, select cluster *j*,**
   together buy or sell, then resolve both clusters.

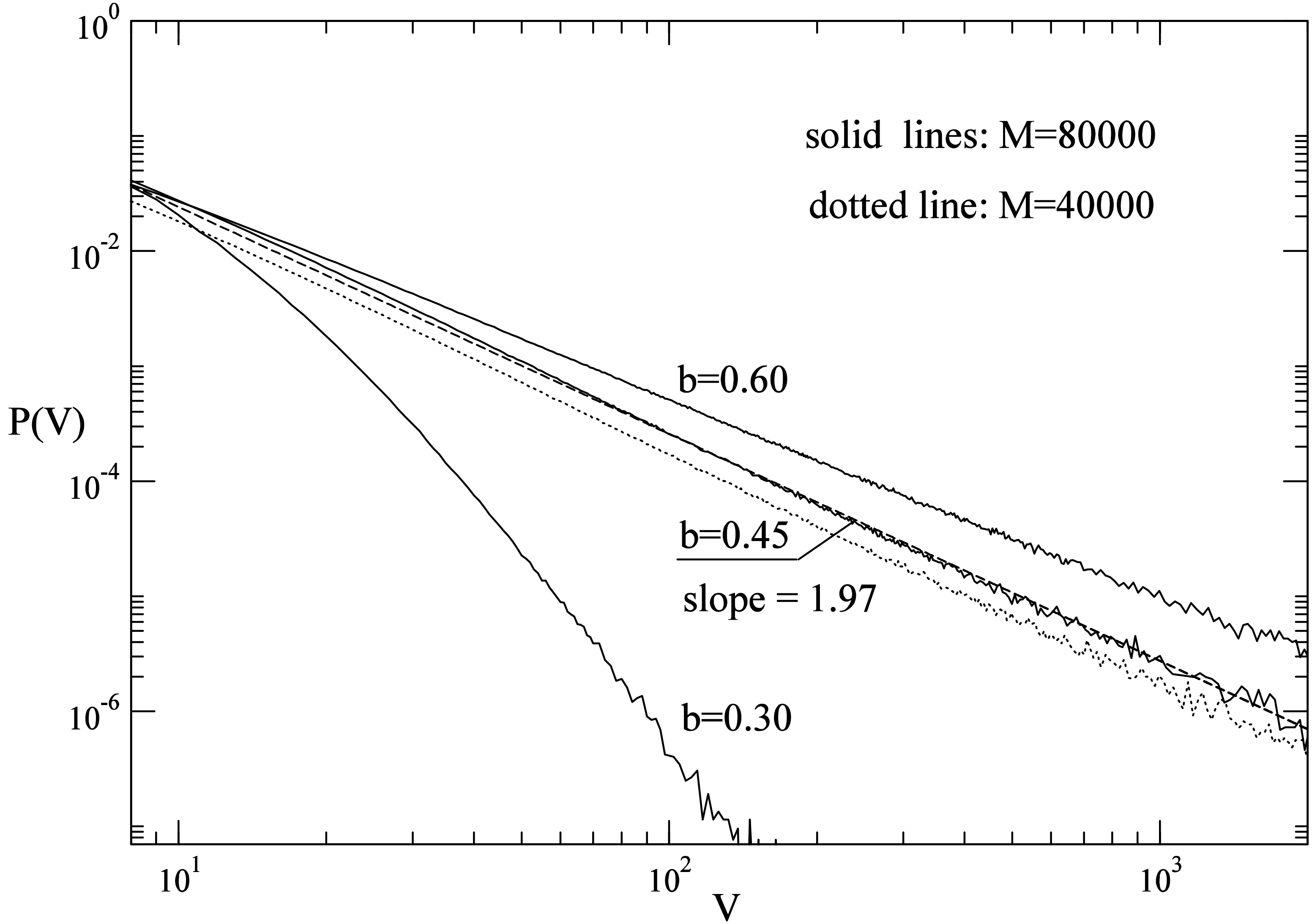
Define ***V(t)*** **as the volume of clusters *i* and *j* ;**

Define Volume imbalance Q(t) as *algebraic sum* of volume *i* and *j*

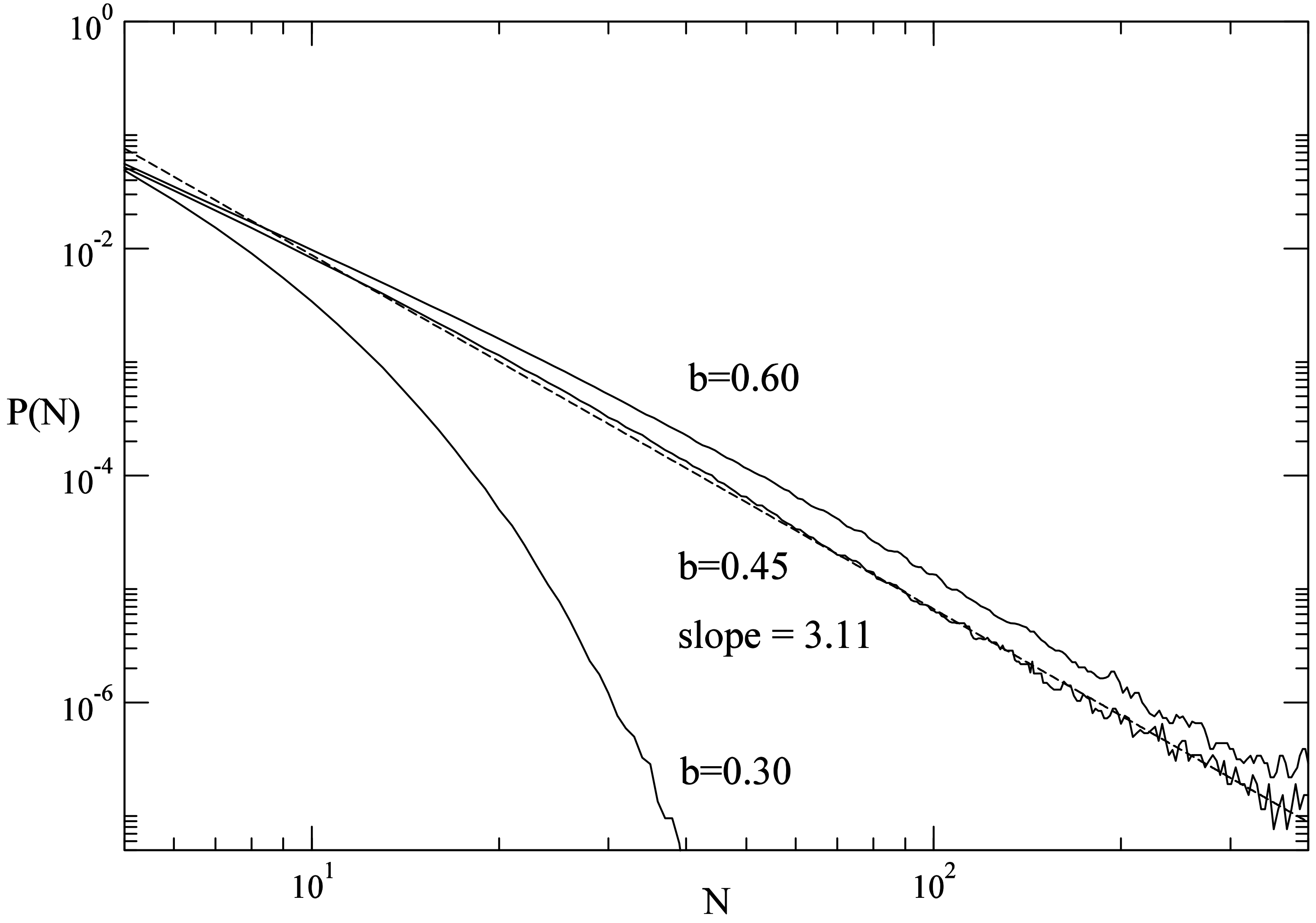
Define price return as

$$r(t) = Sign(Q(t))\frac{\sqrt{|Q(t)|}}{\sqrt{|Q(t)|+A}}\sqrt{N(t)}.$$

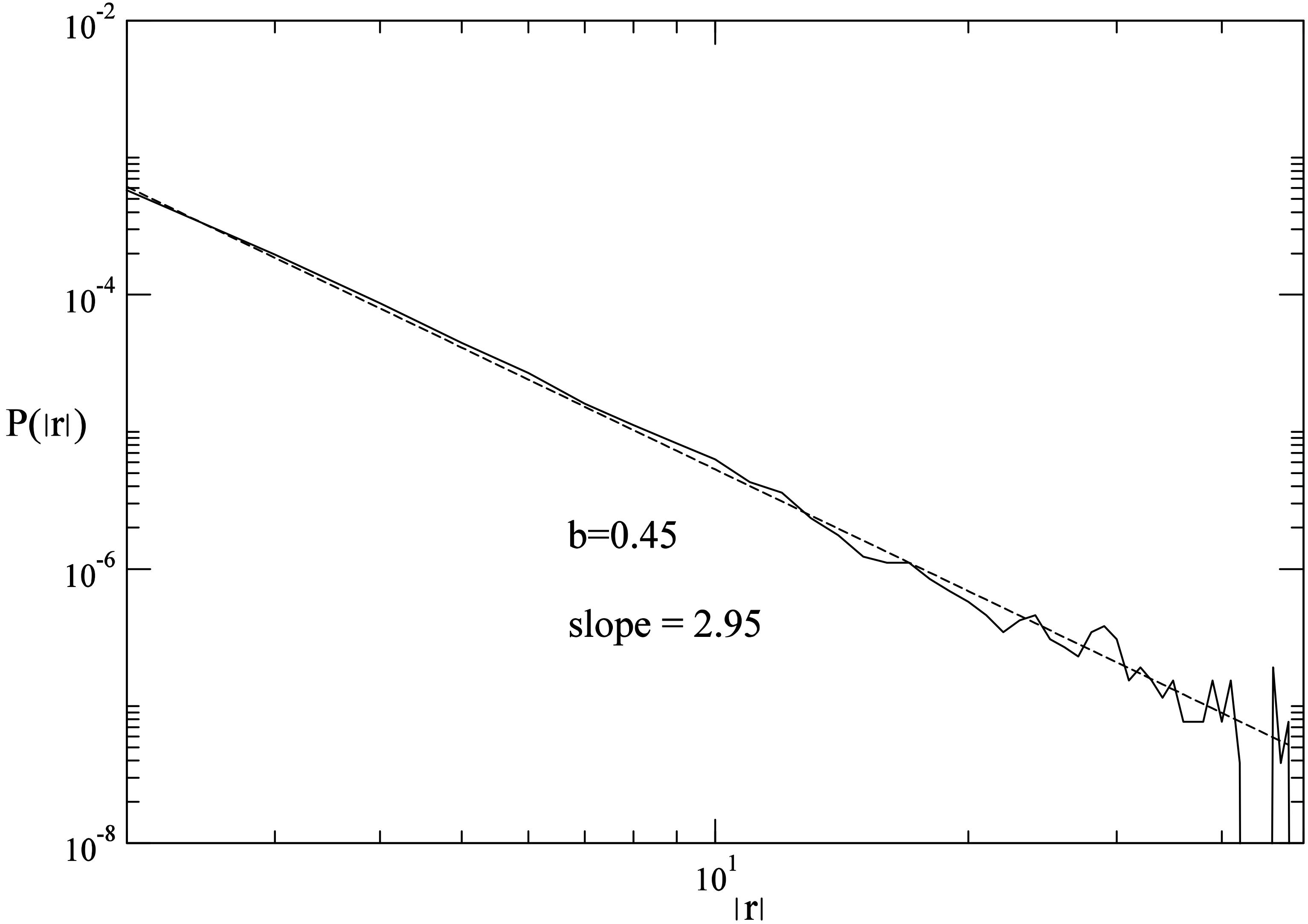
It comes from empirical results.



solid lines: M=80000
dotted line: M=40000
b=0.60
b=0.45
slope = 1.97
b=0.30
P(V)
V
10^0
10^{-2}
10^{-4}
10^{-6}
10^1
10^2
10^3



10^0
10^-2
10^-4
10^-6
P(N)
b=0.60
b=0.45
slope = 3.11
b=0.30
10^1
10^2
N



10^{-2}
10^{-4}
10^{-6}
10^{-8}
P(|r|)
b=0.45
slope = 2.95
10^{1}
|r|

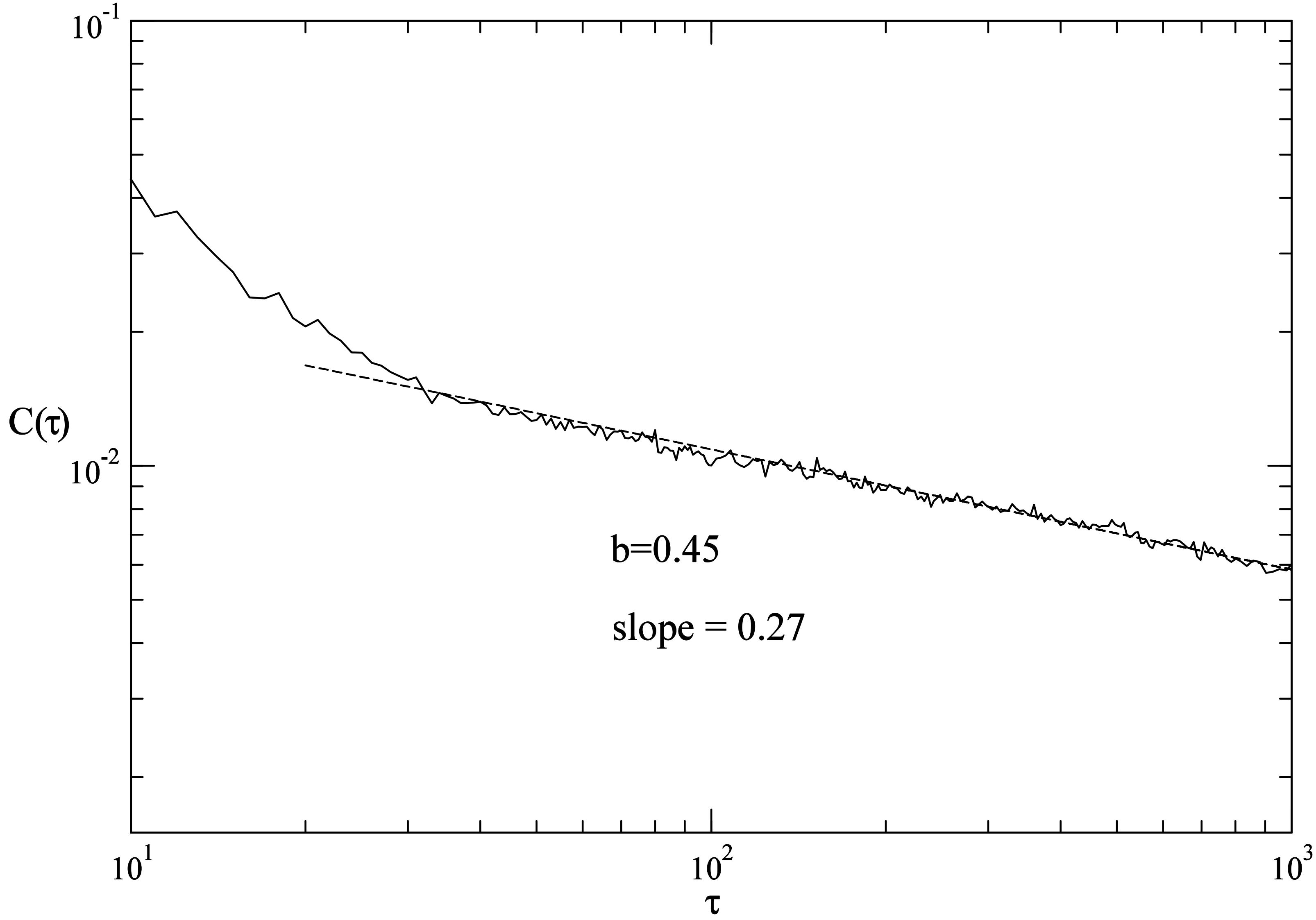
Qualitatively explain the relation among price return, trading volume and number of trades in

Gabaix, Gopikrishnan, Plerou and Stanley, Nature 423, 267 (2003)

$$|r(t)| \sim N(t) \sim \sqrt{V(t)}$$

Auto-correlation function of *r(t)*

$$C(\tau) \equiv \frac{<|r(t)||r(t+\tau)|> - (<|r(t)|>)^2}{<|r(t)|^2> - (<|r(t)|>)^2}$$



C(τ)
10^-1
10^-2
10^1
10^2
10^3
τ
b=0.45
slope = 0.27

- **An interacting dynamic herding model looks reasonable in reproducing the stylized facts of the financial dynamics**
- **How to model the Leverage effects local and non-local in time with the herding model remains open**

Contents