Supplemental Material for

Observation of alternately localized Faraday waves in a narrow tank

I. LASER RANGEFINDER MEASUREMENTS FOR AMPLITUDE AND FREQUENCY OF THE VIBRATOR

The motion of the vibrator can also be measured from a laser rangefinder (Keyence IL-S025). The results are shown in Fig. 1 in this response. With the input parameter $A = 0.018$ cm and $f = 8.66$ Hz, the local maxima of the oscillation of the vibrator varies around $A = 0.018$ cm with a standard deviation of 2.163×10^{-4} cm, leading to a relative error of 1.2%. The power spectrum of the measured oscillation shows a clean peak around $f = 8.66$ Hz, whose exact value is 8.668 Hz, resulting in a relative error of 0.09%. The weight (Spectral Purity) of the power spectrum at $f = 8.668$ Hz is 0.80. We have added the above discussion to the revised manuscript.

II. RELATIVE WEIGHT OF SURFACE TENSION

The surface tension γ will affect the boundary of the instable region in the parameter space. In particular, the dissipation μ shifts the boundary, especially the tip, leftwards or rightwards; and the surface tension γ shifts the boundary upwards and downwards. By adjusting their values simultaneously, one can fit with the experimental data well, as shown in Fig. 8 of the paper, to derive their values. We have listed the value of γ obtained in this way in Table 1. We have specified clearly in Sec. VI A that how this is done and what is the estimation error for these key parameters.

It is known that the waves will redistribute the surfactant, leading to surface tension gradients. To estimate the effect of redistribution of the surfactant, let us first consider the relative effect of the surface tension term, comparing with the leading term caused by gravity.

According to the latest *Release on Surface Tension of Ordinary Water Substance*[1], the surface tension of pure water at $20°C$ is about 72.74 dyn/cm. We have added several drops of Kodak Photo-Flo surfactants in the pure water to decrease the surface tension, such

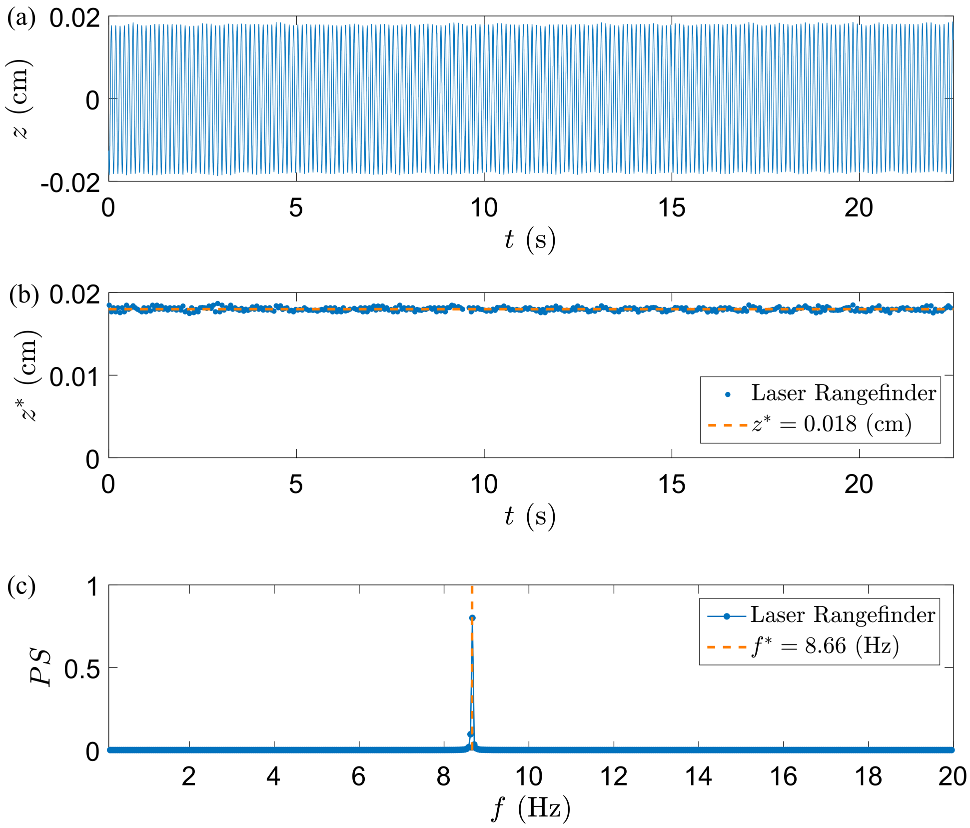

FIG. 1. (a) The z-direction displacement $z(t)$ of the vibrator measured by the laser rangefinder. The driven amplitude and frequency are set as $A = 0.018$ cm and $f = 8.66$ Hz, respectively. (b) The local maxima z^* of $z(t)$, which are distributed around $A = 0.018$ cm. The standard derivation of the local maxima is estimated as 2.163×10^{-4} cm, and the relative error is 1.2%. (c) The power spectra (PS) of the Fourier transform of $z(t)$. The peak is located at $f^* = 8.668$ (Hz). The relative error of frequency is estimated as 0.09%. The weight (Spectral Purity) of the power spectrum at $f = 8.668$ Hz is 0.80.

that it will be smaller than 72.74 dyn/cm. In fact, by fitting to the experimental data, we estimated the surface tension to be 20.0 dyn/cm with a relative error 7.2%. The relative effect of the surface tension can be approximated by the ratio, denoted by $\Gamma_{m,n}$, of the two terms in Eq. (9) in the paper:

$$\Gamma_{m,n} = \frac{\gamma k_{m,n}^2}{g\rho}, \tag{1}$$

where $g = 980.0$ cm/s^2 is the gravity constant, $\rho \simeq 1.0$ g/cm^3 is the mass density of pure water. Regarding to the relevant modes, (12,0) and (8,1), the square of their wave numbers are $k_{12,0}^2 \simeq 0.5685$ cm^{-2} and $k_{8,1}^2 \simeq 0.5789$ cm^{-2}, respectively. So

$$\Gamma_{12,0} \approx \frac{20 \times 0.5685}{980 \times 1} = 0.0116, \quad \Gamma_{8,1} \approx \frac{20 \times 0.5789}{980 \times 1} = 0.0118, \tag{2}$$

which reveals that the effect of the surface tension on modes $(12, 0)$ and $(8, 1)$ is two orders smaller than gravity.

In the case where the surfactant is redistributed by the wave, the maximum effect of the surface tension will be the one for pure water, where $\gamma = 72.74$ dyn/cm. Then

$$\Gamma_{12,0} \approx \frac{72.74 \times 0.5685}{980 \times 1} = 0.0422, \quad \Gamma_{8,1} \approx \frac{72.74 \times 0.5789}{980 \times 1} = 0.0430, \tag{3}$$

which is still much smaller than gravity. Therefore it may have effects, but not a dominant factor for the physics observed here. Indeed, the effect of surface tension cannot be neglected completely. It can shift the unstable region upwards in the parameter space, and through which its value can be estimated.

III. UNCERTAINTY ESTIMATION OF $\beta_{12,0}, \beta_{8,1}$ AND $\alpha_{12,0}$

The dissipation term in the Mathieu equations cannot bound the exponential growth of the amplitude to be finite, in fact, it is the nonlinear term, i.e. $\beta_{12,0}$ and $\beta_{8,1}$, that bound the amplitude. In the meantime, the parameter $\alpha_{12,0}$ plays the role of feedback from the passive mode, so it may also affect the leading mode's amplitude. We can use the asymptotic amplitudes to determine the nonlinear factors $\beta_{12,0}$, $\beta_{8,1}$, and the feedback coupling $\alpha_{12,0}$.

In this framework, we choose a representative pair of parameter in phase space, e.g. $f = 8.66$ Hz, $A = 0.018$ cm. The detailed methodology is as follows. Since there are three coefficients, for each set of values for these three coefficients, we obtain the steady state solutions $\eta_{m,n}$ for mode $(12, 0)$ and $(8, 1)$. Then we fix two of the coefficients, and vary the remaining one. For example, we can fix $\beta_{8,1}$ and $\beta_{12,0}$, and vary $\alpha_{12,0}$. By comparing their amplitudes with that of the experimental data, see Fig. 2(a), we can obtain the amplitude-difference for the two modes $\Delta\eta_{12,0}$ and $\Delta\eta_{8,1}$, as shown in Fig. 2(b) for a representative case. This procedure can be carried out for the other two coefficients. After a few rounds, a set of optimal values for these three coefficients can be obtained. However, since there are still discrepancies between the model and the experiment, a complete simultaneous

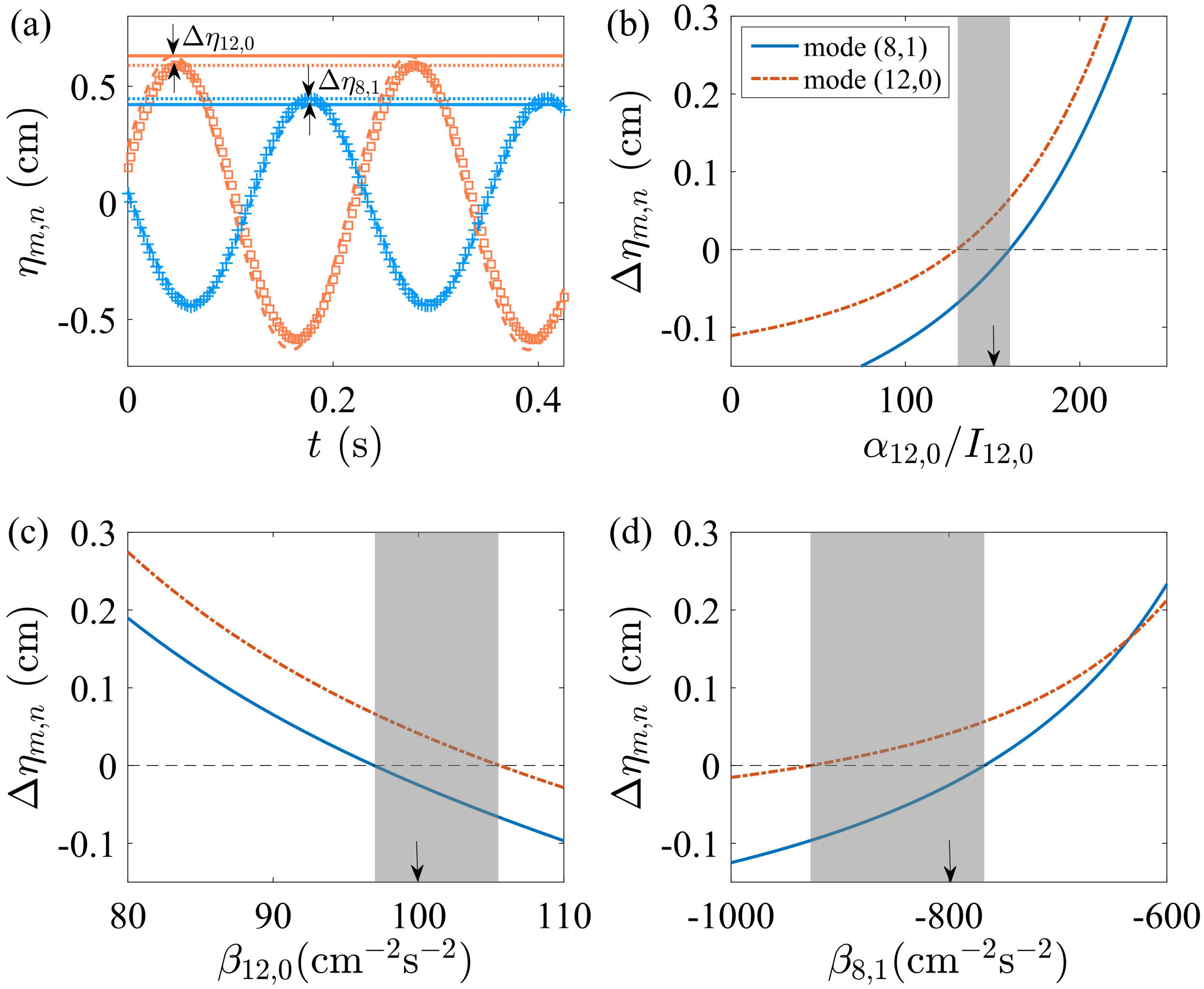

FIG. 2. (a) For the set of the optimal values of the three coefficients, $\alpha_{12,0} = 150/I_{12,0}$, $\beta_{12,0} = 100$ $\mathrm{cm}^{-2}\mathrm{s}^{-2}$, $\mu_{8,1} = 0.65$ s^{-1}, a comparison between the experimental data and the simulation result. (b-d) Fix two of the coefficients at the optimal value, the dependence of $\Delta\eta_{m,n}$ versus the third coefficient: (b) $\alpha_{12,0}$, (c) $\beta_{12,0}$, and (d) $\beta_{8,1}$. The arrows indicate the optimal value. The boundary of the shaded region is when $\Delta\eta_{12,0}$ or $\Delta\eta_{8,1}$ equals to zero. The relative uncertainties are $u_{\alpha_{12,0}} = 10\%$, $u_{\beta_{12,0}} = 4.3\%$, $u_{\beta_{8,1}} = 9.9\%$.

coincidence for both of the modes cannot be obtained. Thus with two coefficients taking the value from the set of optimal coefficients, varying the rest one, there will be two values that each corresponds to a coincidence with one mode, as shown in Fig. 2(b-d). The difference between these two values can be regarded as the uncertainty for this parameter, e.g., for a coefficient p,

$$u = \Delta p/2p_0.$$

From the results in Fig. 2(b-d), we have $u_{\alpha_{12,0}} = 10\%$, $u_{\beta_{12,0}} = 4.3\%$, $u_{\beta_{8,1}} = 9.9\%$.

IV. VIDEO 1

This video shows the surface wave in Figure 2 with A=0.018 cm, f=8.66 Hz.

V. VIDEO 2

This video shows the evolution of the ALFW in Figure 5 with A=0.04 cm, f=8.64 Hz.

REFERENCES

[1]T. Petrova and R. Dooley, Proceedings of the International Association for the Properties of Water and Steam, Moscow, Russia , 23 (2014).