Promoting collective motion of self-propelled agents by distance-based influence

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We propose a dynamic model for a system consisting of self-propelled agents in which the influence of an agent on another agent is weighted by geographical distance. A parameter α is introduced to adjust the influence: The smaller value of α means that the closer neighbors have a stronger influence on the moving direction. We find that there exists an optimal value of α leading to the highest degree of direction consensus. The value of optimal α increases as the system size increases, while it decreases as the absolute velocity, the sensing radius, and the noise amplitude increase.

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I. INTRODUCTION

Collective motion is a ubiquitous phenomenon in nature, examples of which include traffic jams [1], bird flocks [2–5], fish schools [6], insects swarms [7,8], bacteria colonies [9,10], pedestrian flows [11–13], and active granular media [14]. In recent years, a variety of efforts have been devoted to modeling the dynamic properties of swarms [15–27]. Vicsek *et al.* proposed a particularly simple but rich model [28]. In the Vicsek model (VM), some self-propelled agents move with the same absolute velocity in a square-shaped cell with periodic boundary conditions. At each time step, every agent updates its direction according to the average direction of the agents' motion in its neighborhood. The neighborhood of an agent i is composed of agent i itself and those agents who fall in a circle of sensing radius that is centered at the current position of i. It has been demonstrated that all agents will converge to the same direction on a macroscopic scale when the density of the system is high and the noise is small enough [29].

The VM and its variations have attracted much attention in the past decade [30–39]. Grégoire and Chaté found that the onset of collective motion in the VM as well as in related models with and without cohesion is always discontinuous [30]. Huepe and Aldana studied intermittency and clustering in the VM [31]. Yang *et al.* considered a power-law distribution of sensing radius that can enhance the convergence efficiency [32]. Li and Wang proposed an adaptive velocity model in which each agent not only adjusts its moving direction but also adjusts its speed according to the degree of direction consensus among its local neighbors [33]. Tian *et al.* discovered that there exists an optimal view angle leading to the fastest direction consensus [36]. Gao *et al.* proposed a restricted angle model that significantly improves the collective motion of self-propelled agents [38]. Schubring and Ohmann proposed a density-independent modification of the VM in which an agent interacts with neighbors defined by Delaunay triangulation [39]. Peruani and Bär demonstrated that the clustering statistics and the corresponding phase transition to nonequilibrium clustering found in many experiments and simulation studies with self-propelled particles with alignment can be obtained by a simple kinetic model [40]. Buscarino *et al.* found that the direction consensus can be improved by long-range interactions [41,42].

In the original VM, all agents in agent i's neighborhood have the same influence on the moving direction of agent i. However, due to the existence of diversity in human society and the animal world, influences of different agents usually are not the same. It has been shown that the heterogeneous influence plays an important role in various dynamics such as the formation of public opinion [43,44] and the evolution of cooperation [45]. In this paper we propose a weighted VM, where the influence of neighbor j on agent i is determined by the geographical distance between the two agents. We set the weight of neighbors to be an exponential function with a tunable parameter α. Interestingly, we find that there exists an optimal value of α leading to the highest degree of direction consensus. The effects of the moving speed, the sensing radius, the system size, and the noise amplitude on direction consensus of the system are also studied.

The paper is organized as follows. In Sec. II we introduce the distance-based influence model. The simulation and discussion are given in Sec. III. A summary is given in Sec. IV.

II. DISTANCE-BASED INFLUENCE MODEL

We consider N agents moving in the two-dimensional plane without periodic boundary conditions [33,46]. Initially agents are randomly distributed on a region of an $L \times L$ rectangle with random directions. Note that this rectangle does not represent the boundary for motion, but only restricts the initial distribution of positions of agents. Each agent has the same absolute velocity v_0 and sensing radius r. At time t, the position of a specific agent i is updated according to

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + v_0 e^{i\theta(t)}. \quad (1)$$

Its direction is updated as

$$e^{i\theta_i(t+1)} = e^{i\Delta\theta_i(t)} \frac{\sum_{j\in\Gamma_i(t)} W_j(t) e^{i\theta_j(t)}}{\left\| \sum_{j\in\Gamma_i(t)} W_j(t) e^{i\theta_j(t)} \right\|}, \quad (2)$$

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where $\Delta\theta_i \in [-\eta,\eta]$ denotes the white noise, $e^{i\theta(t)}$ is a unit directional vector, $\Gamma_i(t)$ is the set of neighbors of agent i defined by the sensing radius r at time step t, and the weight $W_j(t)$ denotes the influence of neighbor j on agent i at time step t. We define $W_j(t)$ as

$$W_j(t) = e^{\alpha d_{ij}(t)}, \tag{3}$$

where $d_{ij}(t) = \|\mathbf{x}_j(t) - \mathbf{x}_i(t)\|$ is the distance between agent i and j at time step t and α is a tunable parameter. For $\alpha > 0$ ($\alpha < 0$), the farther (closer) neighbors have a larger weight of influence. When $\alpha = 0$, our model is the same as the standard VM, where each agent in the system has the same weight.

III. SIMULATION AND DISCUSSION

In all the following simulations, we set $L = 10$.

We first consider the case in which the noise is zero ($\eta = 0$). From the perspective of complex network theory, the topology of the self-propelled agent system can be expressed as a temporal network [47]. At time t, each agent is represented by a node and an edge between agent i and j is established if the distance between them is shorter than the sensing radius r. A cluster is a subgraph in which any two nodes are connected to each other by paths running along edges of the network. In the case of zero noise, after a period of evolution, agents aggregate into different moving polar clusters. All agents within a moving polar cluster move in the same direction, as shown in Fig. 1.

Following the previous studies [10,48], we examine the cluster size distribution. The results are shown in Fig. 2. We see the probability $P(S)$ that a cluster with size S decays as a power law, following $P(S) \sim S^{-\mu}$. The inset of Fig. 2 shows the dependence of μ on α when the absolute velocity $v_0 = 1$ and the sensing radius $r = 1.6$. We find that the exponent μ is minimum at $\alpha \approx 4$. Since smaller μ indicates a higher probability of large clusters, $\alpha \approx 4$ is an optimal value for the system to have large clusters. The value of μ is found to be in the range $0.8 < \mu < 1.5$, in line with the previous experiments and simulations [10,40,49].

Following the previous studies [32,33,38], we measure the degree of direction consensus by the relative size s_1 of the largest cluster, which is defined as the ratio of the number of agents within the largest cluster to the total number of agents.

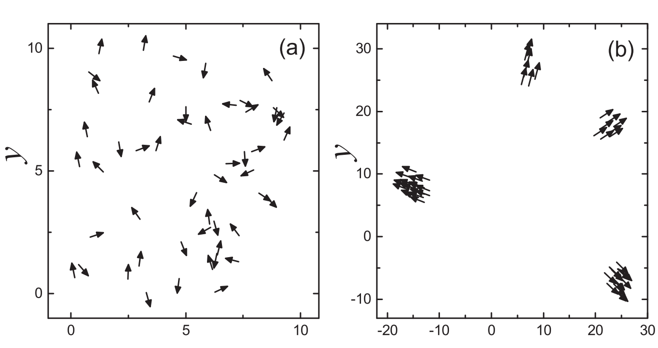

FIG. 1. Snapshots of locations and velocities (a) in the initial configuration and (b) at the time step $t = 100$. The arrows show the direction of motion of the agents. The parameters are set as $L = 10$, $N = 50$, $r = 2$, $v_0 = 0.2$, and $\alpha = 0$.

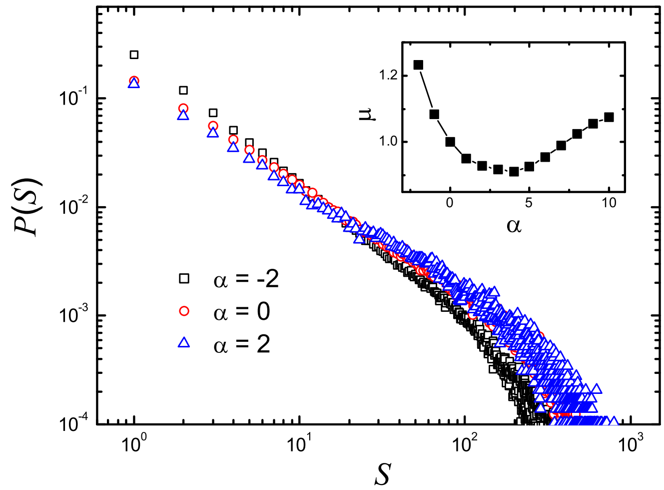

FIG. 2. (Color online) Distribution of cluster sizes $P(S)$ for different values of α. It is found that $P(S) \sim S^{-\mu}$. The inset shows the exponent μ as a function of α. The absolute velocity $v_0 = 1$, the sensing radius $r = 1.6$, and the system size $N = 1000$. Each data point results from an average over 10^4 different realizations.

Note that $0 \leqslant s_1 \leqslant 1$. A larger value of s_1 indicates a higher degree of direction consensus.

Figure 3 shows s_1 as a function of the absolute velocity v_0 for different values of α. From Fig. 3 we can see that for any given value of α, s_1 decreases monotonically as v_0 increases. Figure 4 plots s_1 as a function of the sensing radius r for different values of α. In Fig. 4 one can observe that for any given value of α, s_1 increases with r. Figure 5 depicts s_1 as a function of the system size N for different values of α. From Fig. 5 we find that for any given value of α, s_1 increases with N.

Figure 6 shows the dependence of s_1 on α for different values of the absolute velocity v_0. In Fig. 6 one can observe a resonancelike behavior. For a given value of v_0, there exists

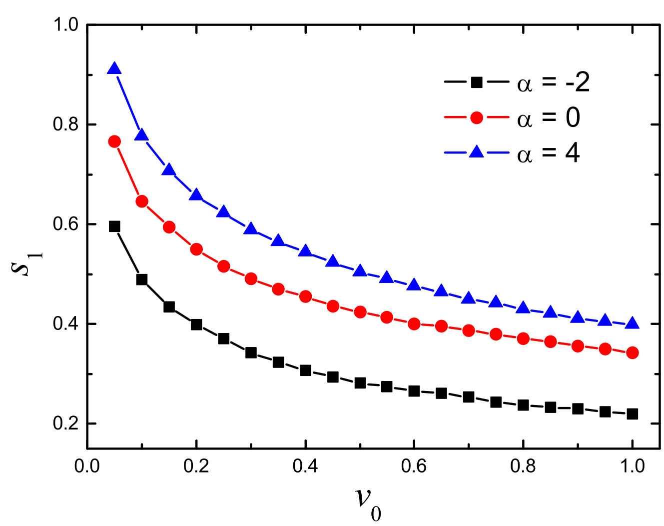

FIG. 3. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of the absolute velocity v_0 for different values of α. The sensing radius $r = 1.6$ and the system size $N = 1000$. Each data point results from an average over 1000 different realizations.

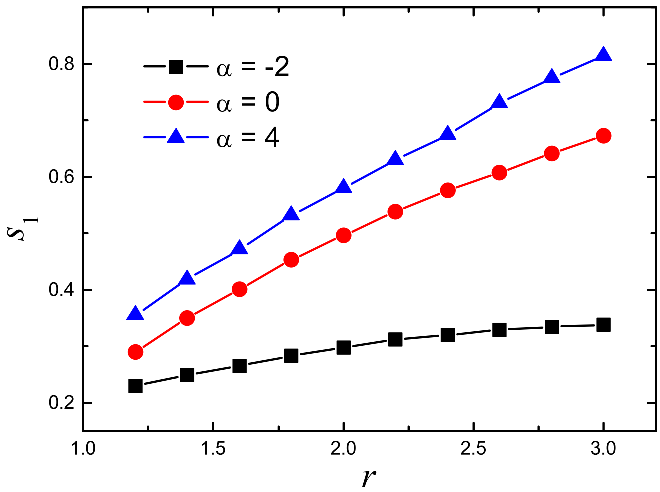

FIG. 4. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of the sensing radius r for different values of α. The absolute velocity $v_0 = 0.6$ and the system size $N = 1000$. Each data point results from an average over 1000 different realizations.

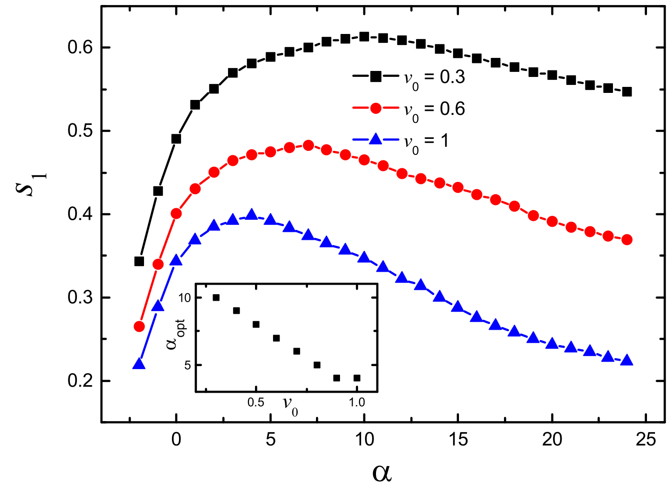

FIG. 6. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of α for different values of the absolute velocity v_0. The inset shows the optimal value of α, $\alpha_{\rm opt}$, as a function of v_0. The sensing radius $r = 1.6$ and the system size $N = 1000$. Each data point results from an average over 1000 different realizations.

an optimal value of α, hereafter denoted by $\alpha_{\rm opt}$, leading to the maximum s_1. It is interesting to note that, for the absolute velocity $v_0 = 1$ and the sensing radius $r = 1.6$, $\alpha_{\rm opt} \approx 4$ not only corresponds to the maximum s_1, but also results in the minimum μ (see the inset of Fig. 2). Intuitively, at this point, a minimum μ corresponds to a distribution $P(S)$ that decreases most slowly as S increases, indicating a larger probability that a large cluster emerges. This implies that the minimum of μ and the maximum of s_1 occur at the same point of $\alpha_{\rm opt} \approx 4$. From the inset of Fig. 6 we find that $\alpha_{\rm opt}$ decreases from 10 to 4 as v_0 increases from 0.3 to 1. Since larger α indicates more influence from distant nodes, this means that as v_0 increases, direction consensus is better preserved by strengthening the influence from closer agents.

Figure 7 shows the dependence of s_1 on α for different values of the sensing radius r. From Fig. 7 we find that for a given value of r, there exists an optimal value of α leading to the maximum s_1. The inset of Fig. 7 shows that $\alpha_{\rm opt}$ decreases from 8 to 3 as r increases from 1.2 to 2.4. Figure 8 shows the dependence of s_1 on α for different values of the system size N. From Fig. 8 one can see that for a given value of N there exists an optimal value of α resulting in the maximum s_1. The inset of Fig. 8 displays that $\alpha_{\rm opt}$ increases from 2 to 5 as N increases from 200 to 10 000.

The resonancelike behavior of the dependence of s_1 on α can be understood as follows. For very small values of α, an agent is mostly influenced by its closest neighbors and the active interaction radius is much shorter than the sensing radius r. As indicated by the $\alpha = -2$ case in Fig. 4, s_1 increases very slowly with r. Previous studies have shown that the direction consensus can be improved by adding long-range interactions

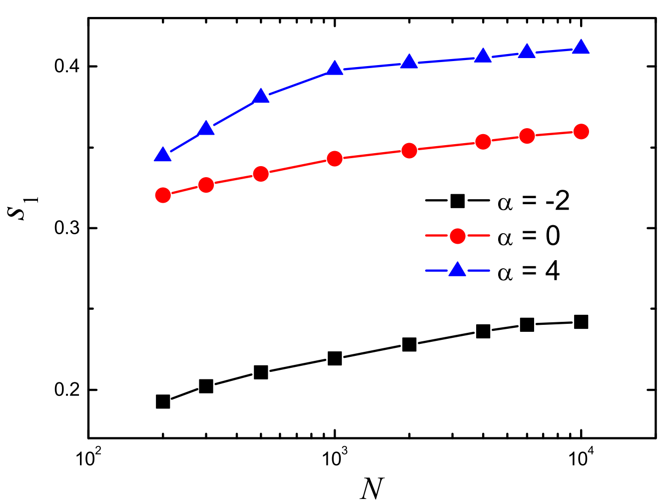

FIG. 5. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of the system size N for different values of α. The absolute velocity $v_0 = 1$ and the sensing radius $r = 1.6$. Each data point results from an average over 1000 different realizations.

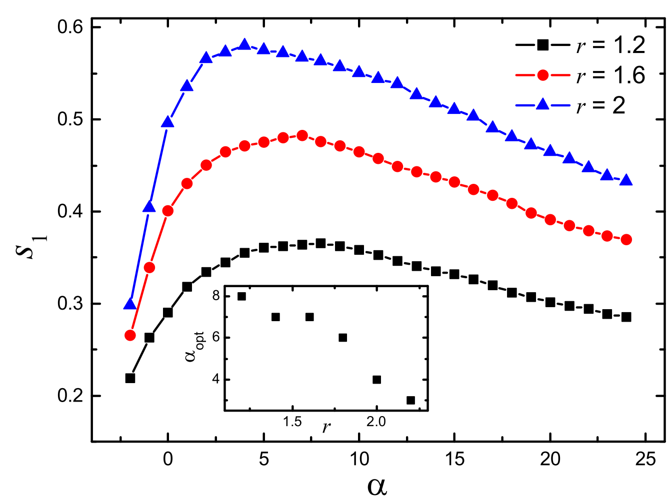

FIG. 7. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of α for different values of the sensing radius r. The inset shows the optimal value of α, $\alpha_{\rm opt}$, as a function of r. The absolute velocity $v_0 = 0.6$ and the system size $N = 1000$. Each data point results from an average over 1000 different realizations.

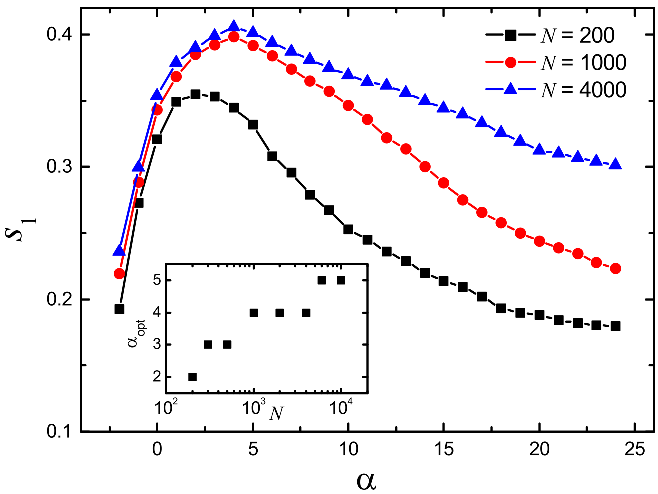

FIG. 8. (Color online) Relative size s_1 of the largest cluster in the steady state as a function of α for different values of the system size N. The inset shows the optimal value of α, α_{opt}, as a function of N. The absolute velocity $v_0 = 1$ and the sensing radius $r = 1.6$. Each data point results from an average over 1000 different realizations.

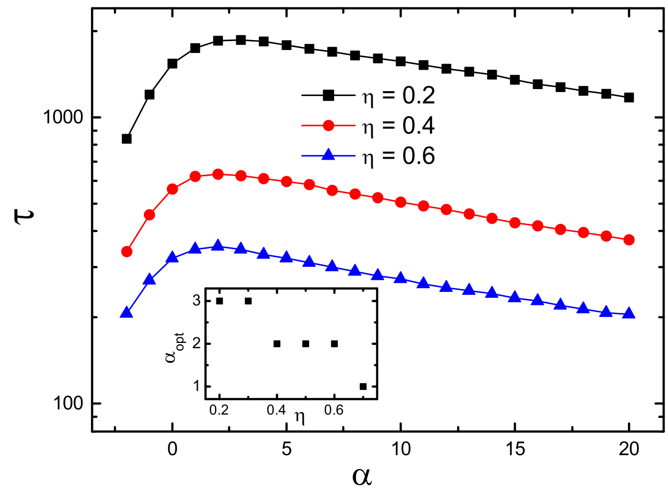

FIG. 10. (Color online) Transient time τ as a function of α for different values of the noise amplitude η. The inset displays the optimal value of α, α_{opt}, as a function of η. The absolute velocity $v_0 = 0.6$, the sensing radius $r = 1.6$, and the system size $N = 1000$. Each data point results from an average over 1000 different realizations.

[41,42,50]. Thus it is beneficial if distant neighbors are given a strong influence. However, when α is too large that an agent is mostly influenced by the farthest neighbor, the interaction between other neighbors would be so weak that is not able to maintain the coherent motion between them, which in turn also disrupts direction consensus. A similar effect has been observed in percolation issues [51]. Therefore, there must be an intermediate value of α that optimizes direction consensus.

Next we study the case in which the noise amplitude η is nonzero. Figure 9 shows s_1 as a function of the time step t for different values of α when $\eta = 0.6$. From Fig. 9 we see that s_1 decreases to 0 as time evolves, indicating that all the agents will disperse without any apparent cluster in the noisy environment. From Fig. 9 one can also observe that the value of α affects the process of dispersion.

To quantify the speed of dispersion in the noisy case, we study the transient time τ, which is defined as the time when s_1 is first below a certain value ϵ. We have checked that qualitative results are invariant when ϵ is small enough. In this paper we take $\epsilon = 0.02$. Figure 10 shows the transient time τ as a function of α for different values of the noise amplitude η. One can see that τ can be maximized by an optimal value of α. This means the process of dispersion can be best slowed down by fine-tuning the value of α. Moreover, the inset of Fig. 10 displays that the optimal value of α, α_{opt} decreases from 3 to 1 as the noise amplitude η increases from 0.2 to 0.7.

In the above studies, the degree of direction consensus of a system is quantified by the relative size of the largest cluster. We have checked that the qualitative results hold unchanged if we use an order parameter to measure the degree of direction

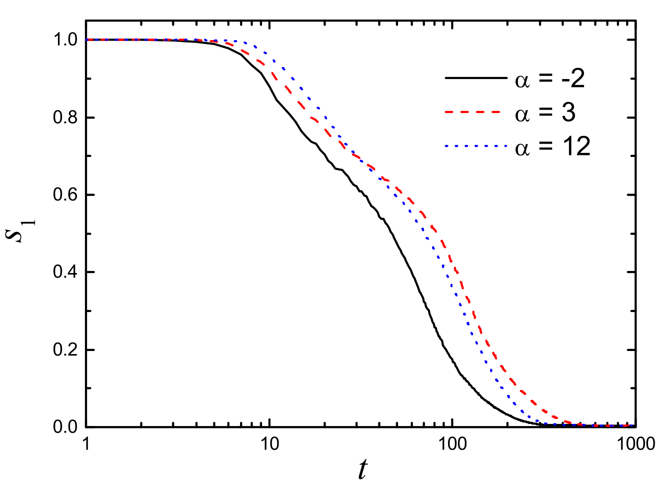

FIG. 9. (Color online) Relative size s_1 of the largest cluster as a function of the time step t for different values of α. The absolute velocity $v_0 = 0.6$, the sensing radius $r = 1.6$, the system size $N = 1000$, and the noise amplitude $\eta = 0.6$. Each curve results from an average over 1000 different realizations.

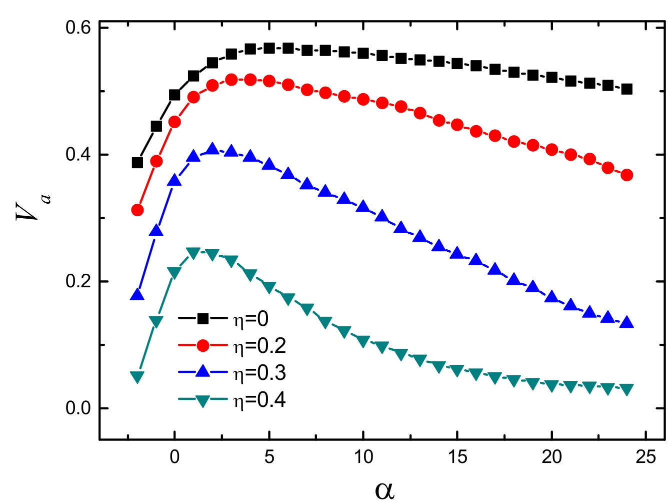

FIG. 11. (Color online) Order parameter V_a as a function of α for different values of the noise amplitude η. The absolute velocity $v_0 = 0.6$, the sensing radius $r = 1.6$, and the system size $N = 1000$. Here V_a is calculated at the time step $t = 500$. Each data point results from an average over 1000 different realizations.

consensus, as defined by [28]

$$V_a = \frac{1}{N}\left\|\sum_{i=1}^{N} e^{i\theta_i(t)}\right\|, \quad 0 \leqslant V_a \leqslant 1. \tag{4}$$

A larger value of V_a indicates a better consensus. Figure 11 shows the dependence of V_a on α for different values of the noise amplitude η. From Fig. 11 one can see that for a given value of η, V_a is maximized by an optimal value of α. In addition, we find that V_a decreases as η increases when α is fixed.

IV. CONCLUSION

We have introduced a weight based on geographical distance into the original Vicsek model. A tunable parameter α is used to govern the weight. The direction of each agent is updated by the weighted average directions of its neighbors. In the noiseless case, self-propelled agents will aggregate into different clusters and the size distribution of clusters exhibits a power-law form. We find that direction consensus is enhanced when the absolute velocity is small and the sensing radius or the system size is large. Interestingly, there exists an optimal value of α that yields the highest level of direction consensus. In the noisy case, agents gradually disperse and each agent moves alone. This process of dispersion can be best slowed down by optimizing α. The value of optimal α depends on the absolute velocity, the sensing radius, the system size, and the noise amplitude. For all kinds of cases, the optimal value of α is always positive, indicating that a suitably strong influence of distant neighbors can best enhance the direction consensus of the system.

Note that recently Gao *et al.* proposed another weighted Vicsek model [46], where the weight of each agent is determined by its number of neighbors. They found that the system more easily achieves direction consensus when agents with more neighbors are allocated greater weight. In the weighted model proposed by Gao *et al.*, agents are assumed to be able to obtain global information, i.e., an agent not only knows the number of its neighbors, but also knows the number of neighbors' neighbors. In our model, an agent only gets local information, i.e., the distance between it and its neighbor. Together Ref. [46] and our work provide a deeper understanding of the impact of a heterogeneous influence on collective motion.

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