Scaling of noisy fluctuations in complex networks and applications to network prediction

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We study the collective dynamics of oscillator-network systems in the presence of noise. By focusing on the time-averaged fluctuation of dynamical variable of interest about the mean field, we discover a scaling law relating the average fluctuation to the node degree. The scaling law is quite robust as it holds for a variety of network topologies and node dynamics. Analyses and numerical support for different types of networks and node dynamics are provided. We also point out an immediate application of the scaling law: predicting complex networks based on time series only, and we articulate how this can be done.

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I. INTRODUCTION

An outstanding topic in nonlinear and statistical physics is concerned about the effect of noise on dynamical systems. Significant phenomena discovered so far include stochastic resonance [1], coherence resonance [2], and noise-induced synchronization [3], etc. Our interest in this paper is in the dynamics of large complex networks *in the presence of noise*. Despite tremendous recent efforts on complex-network dynamics, the issue of noise has been somewhat overlooked. The presence of noise is, however, ubiquitous in realistic, physical, and other natural systems. Since our dynamical system consists of a large number of oscillatory units interacting with each other in a complicated manner, it is meaningful to define, for any given time, a mean field $\langle x\rangle_E$ based on some dynamical variable of interest, say $x(t)$, where $\langle\cdot\rangle_E$ stands for "space" average over the network elements. For node i, because of dynamical evolution under noise, in time its corresponding dynamical variable will fluctuate about the mean field. The average fluctuation over a long observational time interval can then be defined: $\Delta x_j^2 \equiv \langle (x_j(t)-\langle x\rangle_E)^2\rangle_T$, where $\langle\cdot\rangle_T$ denotes the time average. The phenomenon that we wish to report here is that this time-averaged fluctuation scales with the degree k_j (the number of links) of the node as

$$\Delta x_j^2 \sim k_j^{-1}. \tag{1}$$

A feature is that this scaling law holds for a variety of network topologies and node dynamics [4,5]. An application of our finding is that, since Δx_j^2 can be calculated purely and efficiently from time series and because of the one-to-one correspondence between Δx_j^2 and the node degree, the scaling law provides an efficient way to predict the node degrees and consequently hub nodes from network whose detailed topology and node dynamics are not known. Thus, our main result, besides being fundamental to nonlinear physics, also addresses a pressing issue of significant practical interest: network prediction based on time series [6].

In Sec. II, we describe the general method that we use to establish scaling law (1). In Sec. III, we provide extensive numerical evidence for Eq. (1). A theoretical derivation of Eq. (1) is offered in Sec. IV. In Sec. V, we articulate a significant application of scaling law (1): prediction of complex networks based on measured time series. Conclusions are presented in Sec. VI.

II. METHOD

An oscillator network of N nodes under noise can be modeled by the following set of coupled stochastic differential equations:

$$\dot{\mathbf{x}}_j = \mathbf{F}(\mathbf{x}_j) - c\sum_{l=1}^{N} G_{jl}\mathbf{H}(\mathbf{x}_l) + \xi_j\mathbf{M}, \tag{2}$$

where $j=1,\ldots,N$, G_{jl} is the coupling matrix, $\mathbf{H}(\mathbf{x})$ is a coupling function, c is the coupling strength, $\mathbf{M}=[1,0,\cdots,0]^T$, and ξ_j denotes noise. To establish scaling law (1), we consider three types of complex-network topologies (scale free [7], small world [8], and random [9]) and the following three representative dynamical processes:

(1) *consensus dynamics* [10]. Consensus problems have a long history in computer science and form the foundation of the field of distributed computing. In a network of agents, for example, "consensus" means reaching an agreement regarding a policy that depends on the states of all agents. A typical consensus dynamics in the presence of noise can be described as $\dot{x}_j = c\sum_{l=1}^{N} A_{jl}(x_l - x_j) + \xi_j$, where $x_j(t)$ is node j's state at time t and A_{jl} is the adjacency matrix of the network.

(2) *Chaotic dynamics*. We use the chaotic Rössler system described by

$$\begin{cases} \dot{x}_j = -y_j - z_j + c\sum_{l=1}^{N} A_{jl}(x_l - x_j) + \xi_j, \\ \dot{y}_j = x + 0.2y_j + c\sum_{l=1}^{N} A_{jl}(y_l - y_j), \\ \dot{z}_j = 0.2 + z_j(x_j - 9.0) + c\sum_{l=1}^{N} A_{jl}(z_l - z_j). \end{cases} \tag{3}$$

(3) *Kuramoto phase oscillators* [11]. The model has been a paradigm for studying biological and physical systems of large numbers of units interacting through their phases: $\dot{\theta}_j = \omega_j + c\sum_{l=1}^{N} A_{jl}\sin(\theta_l - \theta_j) + \xi_j$, where θ_j and ω_j are the phase and the natural frequency of oscillator j.

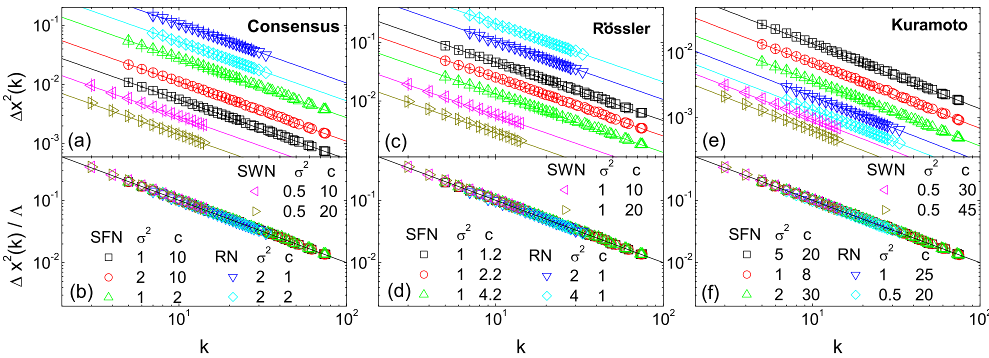

FIG. 1. (Color online) Average fluctuation $\Delta x^2(k)$ as a function of the node degree k for different values of noise variance σ^2 and coupling strength c on scale-free, random, and small-world networks for (a) consensus dynamics, (c) Rössler dynamics, and (e) Kuramoto dynamics. (b), (d), and (f) Rescaled quantity $\Delta x^2(k)/\Lambda$, where $\Lambda=\sigma^2(1+1/\langle k\rangle)/(2c)$, versus k for consensus, Rössler and Kuramoto dynamics, respectively. Data points are from a single network configuration, and $\Delta x^2(k)$ is obtained by averaging over all nodes of degree k with error bars. The parallel lines in (a), (c), and (e) are theoretical predictions from Eq. (13), and the lines in (b), (d), and (f) are the function $1/k$. Network size is 500. For the scale-free network, the lowest degree is $k_{\min}=5$. For the random network, the connection probability among nodes is 0.03. For the small-world network, the average degree is 8 and the rewiring probability is 0.1. The natural frequency ω_i in the Kuramoto model is chosen independently from a prescribed probability distribution $g(\omega)=3/4(1-\omega^2)$ for $|\omega|\le 1$ and $g(\omega)=0$ otherwise.

III. NUMERICAL RESULTS

A. Model networks

We begin by presenting numerical evidence for Eq. (1). Figure 1(a) shows the scaling of $\Delta x^2(k)$ with k for consensus dynamics for different types of networks under noise of various amplitudes, where $\Delta x^2(k)$ is computed after the system settles in a steady state. We observe that, regardless of the network topology and of the noise strength, on a logarithmic scale all scaling curves fall on parallel straight lines of slope -1. After a proper rescaling of $\Delta x^2(k)$ (as suggested by our theoretical analysis below), all data collapse into the single curve $1/k$. Essentially the same results have been obtained for the chaotic Rössler dynamics and for the Kuramoto model, as shown in Figs. 1(c) and 1(e), respectively. All these suggest strongly the existence of scaling law (1).

B. Real-world networks

Since realistic networks may possess topological properties that are not captured by model networks, we set out to test whether scaling law (1) holds for real-world networks. In particular, we investigate the three types of node dynamics on six real-world networks from social, biological, and technological contexts. Simulation results are shown in Figs. 2(a)–2(c). Again, the rescaled average fluctuations collapse to the curve $1/k$, indicating the applicability of Eq. (1) to real-world networks.

C. Clustered network and nonidentical oscillators

We also test the applicability of the scaling law for clustered networks with community structures and coupled nonidentical oscillators. In the community networks, nodes are densely connected within a community but there are sparse links between communities. Simulation results are carried out by adopting the identical Rössler systems on community networks. For the nonidentical oscillators, we choose the nonidentical Rössler systems as an example to make a comparison with the identical oscillators. The nonidentical Rössler system is described as follows:

$$\begin{cases} \dot{x}_i = -\omega_i y_i - z_i + c\sum_{j=1}^{N} A_{ji}(x_j - x_i) + \xi_i \\ \dot{y}_i = \omega_i x_i + 0.2 y_i + c\sum_{j=1}^{N} A_{ji}(y_j - y_i) \\ \dot{z}_i = 0.2 + z_i(x_i - 9.0) + c\sum_{j=1}^{N} A_{ji}(z_j - z_i) \end{cases}, \tag{4}$$

where ω_i governs the natural frequency of an individual oscillator and ω_i for an arbitrary node i is randomly chosen from a range $[a_1, a_2]$. As shown in Fig. 3. One can still find the scaling law for both community networks and noniden-

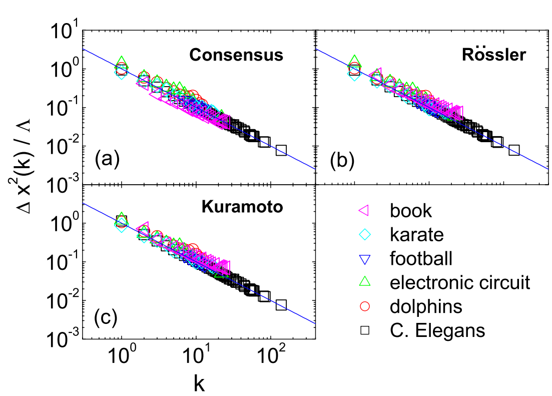

FIG. 2. (Color online) For six different types of real-world networks and the three types of node dynamics (a) consensus, (b) Rössler, and (c) Kuramoto considered in this paper; rescaled quantity $\Delta x^2(k)/\Lambda$ versus k. The blue line is the rescaled theoretical result $1/k$. The real-world networks are from social, biological, and technological contexts. In particular, they are: (1) network of political book purchases (book) [14]; (2) social network of friendships of karate club (karate) [15]; (3) network of American football games among colleges (football) [16]; (4) electron circuit networks (electronic circuit) [17]; (5) dolphin social network (Dolphins) [18]; and (6) the neural network of C. Elegans [8].

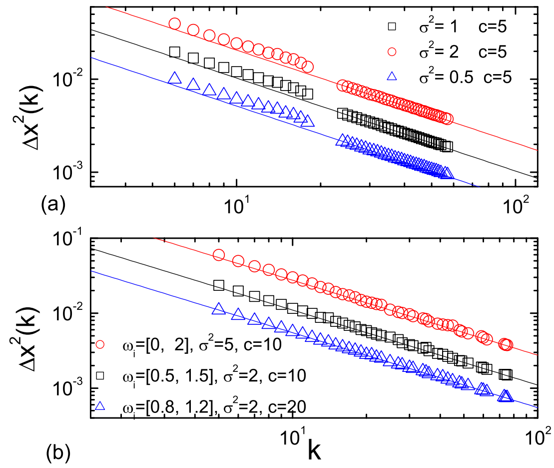

FIG. 3. (Color online) $\Delta x^2(k)$ as a function of degree k for (a) a clustered network of identical Rössler oscillators and (b) nonidentical Rössler oscillators on a scale-free network. The clustered network has two clusters of different sizes: one contains 400 nodes and another has 100 nodes. The connection probability within each cluster is 0.1, and that between nodes from different clusters is 0.001. The scale-free network is the same as in Fig. 1. The lines are theoretical predictions from Eq. (13).

tical Rössler systems with good agreement with the theoretical prediction.

IV. THEORY

A. Unweighted networks

We now provide an analytic theory for scaling law (1). For the consensus dynamics, the variational equation about the consensus state is

$$\Delta\dot{x}_j = -ck_j\Delta x_j + c\sum_{l=1}^{N} A_{jl}\Delta x_l + \xi_j, \qquad (5)$$

where $\Delta x_j = x_j - \langle x\rangle$ and $\langle x\rangle = (1/N)\Sigma_{l=1}^{N} x_l(t)$. Under noise, $\Delta x_l(t)$ is also a random variable. However, comparing to the Gaussian noise ξ_j, we numerically find that the contribution of the summation term to the fluctuation of Δx_j is negligible. Therefore, we shall first neglect this term to obtain the zeroth-order approximation for $\Delta x_j(t)$ [or $\Delta x_l(t)$]. The result can then be used to obtain the first-order correction due to the summation term. Neglecting this term in Eq. (5) leads to

$$\Delta\dot{x}_j + ck_j\Delta x_j = \xi_j. \qquad (6)$$

This is a linear stochastic differential equation. Regarding ξ_j as the input and Δx_j as the output, the transfer function is given by

$$H_j(2\pi i f) = \frac{1}{2\pi i f + ck_j}. \qquad (7)$$

The power spectral densities (PSDs) of the output Δx_j can then be written as

$$S_{\Delta x_j}(f) = |H_j(2\pi i f)|^2 S_{\xi_j}(f), \qquad (8)$$

where i is the imaginary unit, and $S_{\xi_j} = \sigma^2$ is the PSD for the Gaussian white noise ξ_j. From numerical observations we find that Δx_l are approximately independent. Denoting $c\Sigma_{l=1}^{N} A_{jl}\Delta x_l$ as $\mathcal{X}_j$, we have,

$$S_{\mathcal{X}_j} \approx c^2\sum_{l=1}^{N} A_{jl}S_{\Delta x_l} = c^2\sum_{l=1}^{N}\frac{A_{jl}\sigma^2}{4\pi^2 f^2 + c^2k_l^2} \approx \frac{c^2k_j\sigma^2}{4\pi^2 f^2 + c^2\langle k\rangle^2}. \qquad (9)$$

Here, the approximation in Eq. (9) to neglect any degree-degree correlation can be considered as a higher-order approximation. Equation (5) can then be rewritten as

$$\Delta\dot{x}_j + ck_j\Delta x_j = \mathcal{X}_j + \xi_j. \qquad (10)$$

Treating $\mathcal{X}_j$ and ξ_j as independent noise inputs, we have $S_{\Delta x_j}(f) = |H_j(2\pi i f)|^2[S_{\mathcal{X}_j}(f) + S_{\xi_j}(f)]$. The variance of output Δx_j is thus given by

$$\langle\Delta x_j^2\rangle_T = \int_{-\infty}^{\infty} S_{\Delta x_j} df = \int_{-\infty}^{\infty} |H(2\pi i f)|^2[S_{\mathcal{X}_j}(f) + S_{\xi_j}(f)]df. \qquad (11)$$

Inserting the results for $S_{\mathcal{X}_j}$ and S_{ξ_j} and integrating Eq. (11), we obtain

$$\langle\Delta x_j^2\rangle_T \approx \frac{\sigma^2}{2ck_j} + \frac{\sigma^2}{2c\langle k\rangle(k_j + \langle k\rangle)}, \qquad (12)$$

where the second term on the right-hand side is the contribution from $c\Sigma_{l=1}^{N} A_{jl}\Delta x_l$. For nodes with large degrees, $k_j \gg \langle k\rangle$, we have

$$\langle\Delta x_j^2\rangle_T \approx \frac{\sigma^2}{2ck_j}\left(1 + \frac{1}{\langle k\rangle}\right). \qquad (13)$$

For nonlinear or chaotic node dynamics, the technique of variational equations can be used. In general, this can be done as follows. Without noise, the dynamical system can be written as $\dot{\mathbf{x}}_j = \mathbf{F}(\mathbf{x}_j) - ck_j\mathbf{x}_j + c\Sigma_l A_{jl}\mathbf{x}_l$, where $\mathbf{x} \equiv [x, y, z]^T$. When oscillators are in the vicinity of the synchronization state, a variational approach yields $\Delta\dot{\mathbf{x}}_j \approx -ck_j\Delta\mathbf{x}_j + c\Sigma_l A_{jl}\Delta\mathbf{x}_l$. Consider, for example, the case where noise is applied to the first component of the Rössler system. We have $\Delta\dot{x}_j = -ck_j\Delta x_j + c\Sigma_l A_{jl}\Delta x_l + \xi_j$, which is identical to Eq. (5). Scaling law (1) should then hold for generic chaotic node dynamics.

For the Kuramoto model, in the vicinity of the synchronization state, the sinusoidal function can be approximated by a linear function: $\sin(\theta_l - \theta_j) \approx \theta_l - \theta_j$. The dynamical system thus becomes $\dot{\theta}_j = \omega_j - ck_j\theta_j + c\Sigma_{l=1}^{N} A_{jl}\theta_l + \xi_j$. The variational equation is given by $\Delta\dot{\theta}_j = -ck_j\Delta\theta_j + c\Sigma_{l=1}^{N} A_{jl}\Delta\theta_l + \xi_j$, which is the same as Eq. (5) for the consensus dynamics. We thus expect scaling law (1) to hold for the nonidentical Kuramoto type of node dynamics as well.

Note that, although a complete synchronization state in the absence of noise is not necessarily required for scaling law (1), all oscillators should be in an approximately synchronous state to ensure similar oscillation modes. For the consensus dynamics, a consensus state (synchronization) can be naturally achieved, regardless of the network structure and the coupling strength. In this case, we expect scaling law (1) to be robust. For nonlinear dynamics such as those give by the Rössler system, the master-stability function provides a sufficient condition for the validity of scaling law (1). For instance, for a V-shape master-stability function, to drive the system to an approximately synchronization state, coupling strengths among nodes should be chosen in a certain range determined by both the master-stability function and the eigenvalues of the Laplacian matrix. Otherwise, for relatively low or high values of the coupling strength, node degrees and fluctuations may be uncorrelated or the system may diverge, respectively. For a complex clustered network, if the clusters are loosely connected and coupling strengths are small, oscillators in different clusters cannot achieve synchronization due to the weak coupling among clusters, and scaling law (1) may not be valid. Analogously, strong heterogeneity in degree distribution precludes synchronization as well, so scaling law (1) can break in strongly heterogeneous networks. However, for nonlinear dynamics, when oscillators' states are reasonably correlated, the variational equation by linearization is applicable. In this case, the derivations for the scaling law can be valid even under a first-order approximation. Indeed, our theoretical predictions for the three types of dynamics agree with the numerical results very well.

B. Weighted networks

In general, we can show that the scaling law is a property of weighted networks associated with the general dynamical system described by Eq. (2). The variational equation is

$$\Delta\dot{\mathbf{x}}_j = D\mathbf{F}(\bar{\mathbf{x}})\Delta\mathbf{x}_j - c\sum_{l=1}^{N} G_{jl}D\mathbf{H}(\bar{\mathbf{x}})\Delta\mathbf{x}_l + \xi_j\mathbf{M}$$

$$= [D\mathbf{F}(\bar{\mathbf{x}}) - cD\mathbf{H}(\bar{\mathbf{x}})s_j]\Delta\mathbf{x}_j - cD\mathbf{H}(\bar{\mathbf{x}})\sum_{l=1,l\neq j}^{N} G_{jl}\Delta\mathbf{x}_l + \xi_j\mathbf{M}, \tag{14}$$

where $s_j = G_{jj}$ is the strength of node j and $\Sigma_{l=1}^{N} G_{ij}=0$. For high-degree nodes and large c, we have $D\mathbf{F}(\bar{\mathbf{x}}) - cD\mathbf{H}(\bar{\mathbf{x}})s_j \approx -cD\mathbf{H}(\bar{\mathbf{x}})s_j$. Focusing on one component (derivations for other components are similar), say Δx_j, we have

$$\Delta\dot{x}_j = -c(D\mathbf{H})_{11}s_j\Delta x_j - c(D\mathbf{H})_{11}\sum_{l\neq j} G_{jl}\Delta x_l + \xi_j, \tag{15}$$

where $(D\mathbf{H})_{11}$ is a constant for linear coupling and the small influences of the couplings from other components have been neglected [12]. Note that this equation has the same form as Eq. (5). We can thus obtain a general scaling law

$$\langle \Delta x_j^2 \rangle_T \approx \frac{\sigma^2}{2c(D\mathbf{H})_{11}s_j}\left(1 + \frac{1}{\langle s \rangle}\right). \tag{16}$$

To be more general, we note that the coupling matrix $\mathbf{G}$ can be weighted: $G_{ij} = -A_{ij}w_{ij}$, where w_{ij} is the edge weight. For unweighted networks, we have $s_j = k_j$. We study both the homogeneous and heterogeneous edge weights to test the theory for weighted networks. The homogeneous edge weight w_{ij} is randomly chosen from a range $[a_1, a_2]$. The heterogeneous edge weight is

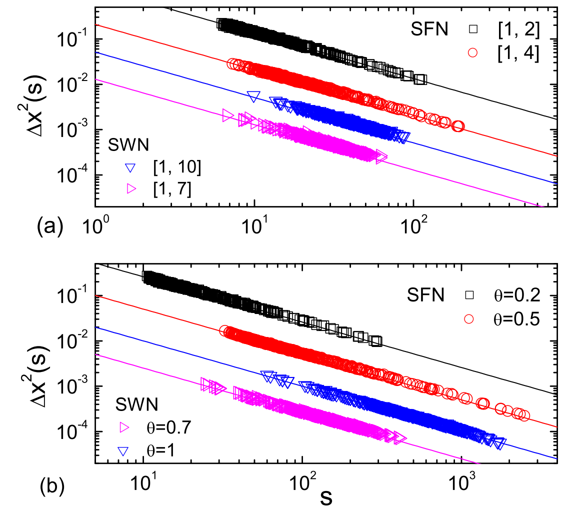

FIG. 4. (Color online) Average fluctuation $\Delta x^2(s)$ as a function of the node strength s for weighted small-world and scale-free networks for (a) homogeneous edge weight w_{ij} randomly chosen from $[a_1, a_2]$ and (b) heterogeneous edge weight $w_{ij} = (k_i k_j)^\theta$, where k_i and k_j are the degrees of nodes i and j. The node dynamics are chosen (quite arbitrarily) to be that of the chaotic Rössler oscillator. The lines are theoretical predictions from Eq. (16). The quantities (σ^2, c) are (10.0,4.0), (2.0,5.0), (1.0,10.0), and (0.5,20.0) (from top to bottom) in (a) and (10.0,2.0), (2.0,2.0), (1.0,5.0), and (0.5,10.0) in (b). The network parameters are the same as in Fig. 1.

$$w_{ij} = (k_i k_j)^\theta, \tag{17}$$

where k_i and k_j are the degrees of nodes i and j. Figure 4 shows examples of general scaling law (16) for both homogeneously and heterogeneously weighted networks, where the node dynamics are that of the chaotic Rössler oscillator. We see that simulation results agree well with the prediction from Eq. (16).

V. APPLICATION: PREDICTING COMPLEX NETWORKS

We shall elaborate a significant application of scaling law (1): network prediction based on time series [13]. The problem can be stated as follows. Given an unknown network and given a set of measured time series from the network, can one infer certain properties of the network based solely on the time series? Our point is that scaling law (1) allows an important characteristic of the network, the node degree (consequently hub nodes), to be detected. Our proposed method is as follows. We first estimate the degree k_l of an arbitrary node (say l). This can be done by disabling any node that is connected to node l. Denote this node by m. When it is disabled, the degree of node l becomes $k_l - 1$ and its average fluctuation becomes

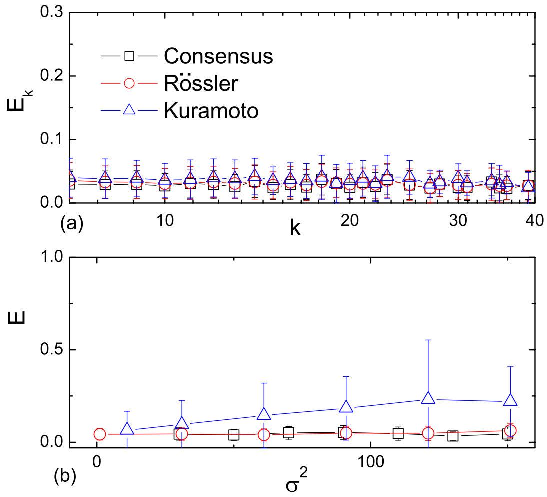

FIG. 5. (Color online) For scale-free networks of size $N=500$ and minimum degree $k_{\min}=5$, prediction error as functions of (a) node degree k and (b) noise variance σ^2 for three node dynamics. Here the prediction error for an arbitrary node i is defined as the absolute value of the difference between the original degree and the predicted degree, normalized by the original degree. Quantities E_k and E are the average over nodes of degree k and the average over all nodes, respectively. All data are obtained by averaging over ten realizations.

$$\Delta x_l'^2 = \frac{\sigma^2}{2c(k_l-1)}\left(1+\frac{1}{\langle k\rangle}\right), \tag{18}$$

which can be measured. It should be emphasized that this can be done without explicit knowledge about the network structure and dynamics. Taking the ratio between the original fluctuation Δx_l^2 and $\Delta x_l'^2$ yields

$$\frac{\Delta x_l^2}{\Delta x_l'^2} = \frac{\sigma^2(1+1/\langle k\rangle)/(2ck_l)}{\sigma^2(1+1/\langle k\rangle)/[2c(k_l-1)]}, \tag{19}$$

which gives

$$k_l = \frac{1}{1-\Delta x_l^2/\Delta x_l'^2}. \tag{20}$$

After k_l has been estimated, the degree of *any* node j in the network can be determined according to scaling law (1),

$$k_j = k_l\frac{\Delta x_l^2}{\Delta x_j^2}. \tag{21}$$

The advantage of this method is that the errors in the predictions of k_i and c can be eliminated. As shown in Fig. 5(a), the prediction error E_k of different degrees for the three types of dynamics is generally less than 5%. The overall prediction error averaged over all nodes is shown in Fig. 5(b). One can see that E for the consensus and for the Rössler dynamics is insensitive to the increase in σ, even large noise variance (e.g., $\sigma^2=200$). For the Kuramoto dynamics, if the noise variance is not too large (say the noise amplitude is of the order of the range of typical dynamical variables), the prediction error is small.

The noise variance can also be estimated from time series. Consider, for example, consensus dynamics on nodes. Summing up equations for all nodes yields

$$\sum_{i=1}^{N}\dot{x}_j = \sum_{i=1}^{N}\xi_j. \tag{22}$$

Denoting $\Sigma_{i=1}^{N}x_j$ as Γ and $\Sigma_{i=1}^{N}\xi_j$ as Λ, we have $\dot{\Gamma}=\Lambda$, where Λ follows a Gaussian distribution but of variance $\sigma_\Lambda^2=N\sigma^2$. The power spectral density of Γ is

$$S_\Gamma(f) = \frac{N\sigma^2}{4\pi^2 f^2}. \tag{23}$$

The variance σ^2 can be obtained by fitting the power spectral density curve to αf^{-2} with $\alpha=N\sigma^2/(4\pi^2)$. With the estimated node degrees and noise variance, the coupling strength c can be estimated from Eq. (13).

VI. CONCLUSIONS

In conclusion, we have presented a scaling law for complex networks: the average fluctuation of a dynamical variable characterizing the node state has a power-law dependence on the node degree. The scaling law holds for a variety of network topologies and node dynamics. We have provided theoretical analysis and extensive numerical evidence to establish the scaling law. As a significant application, the scaling law can be used to address the important but challenging problem of network prediction. In particular, based on a set of measured time series only, the scaling law allows the node degree and a set of hub nodes to be predicted in a computationally efficient way.

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