Agent-based model with asymmetric trading and herding for complex financial systems

Important features

- fat-tail distribution of returns
- long-term correlation of volatilities
- negative and positive return-volatility correlations

Return-volatility correlation

Negative return-volatility correlation: leverage effect, which implies that past negative returns increase future volatilities. Almost all stock markets.

Positive return-volatility correlation: the anti-leverage effect.
Chinese stock markets, 1990-2003

Return-volatility correlation

price: $Y(t)$

return: $R(t) = \ln[Y(t)/Y(t-1)]$

volatility: $|R(t)|$

normalized return: $r(t) = [R(t) - \langle R(t) \rangle]/\sigma$

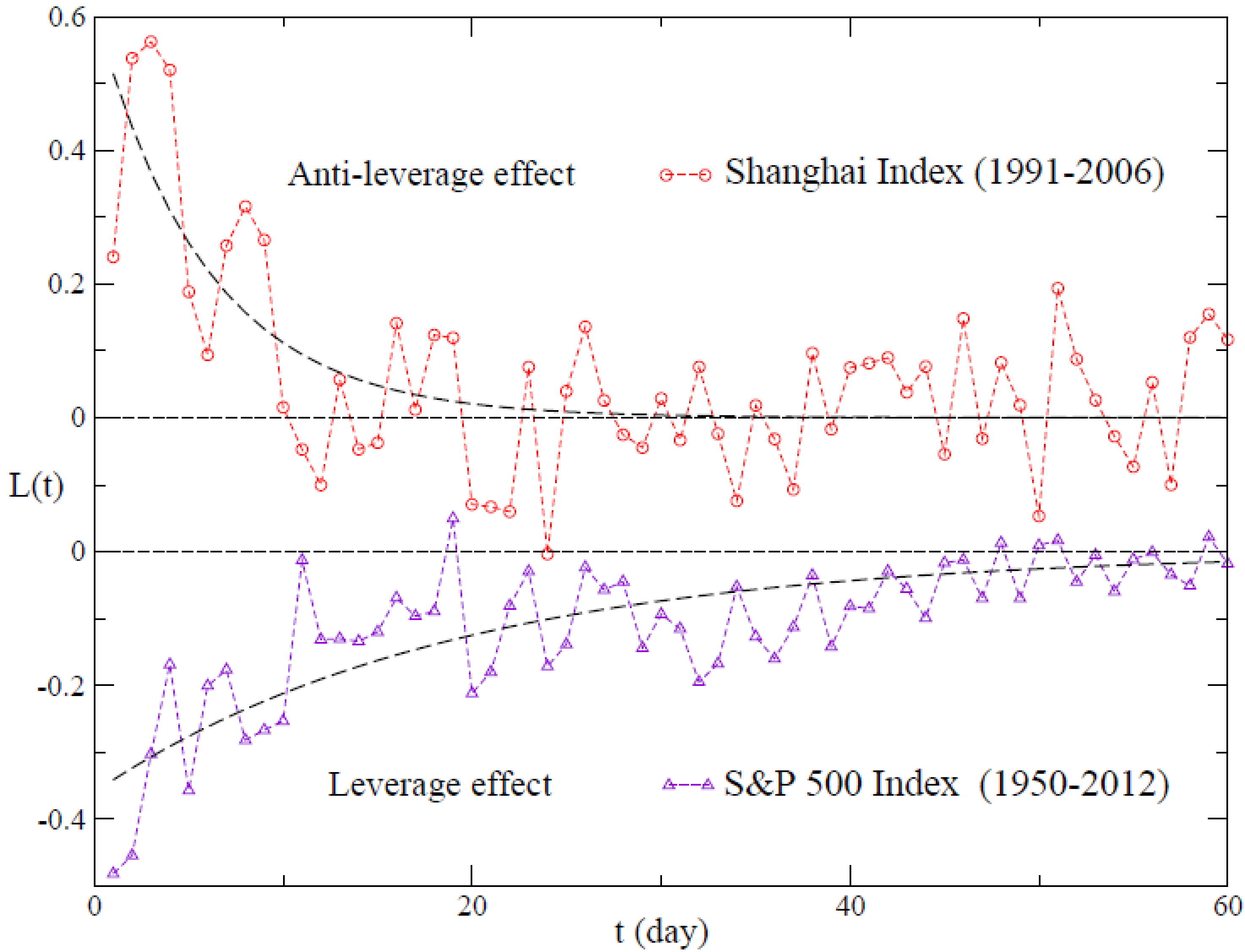
return-volatility correlation function:

$$L(t) = [\langle r(t') \cdot |r(t'+t)|^2 \rangle - L_0]/Z$$

auto-correlation function of volatilities:

$$A(t) = [\langle |r(t')||r(t'+t)| \rangle - \langle |r(t')| \rangle^2]/A_0$$

Return-volatility correlation



Motivation

- price dynamics
- risk management
- optimal portfolio choice

- The microscopic origination of the leverage and anti-leverage effects is still not understood
- How to produce these effects in agent-based modeling remains open

Conception

- determine model parameters from empirical data rather than from statistical fitting of the results

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Linking agent-based models and stochastic models of financial markets

Ling Feng[a,b,c,1], Baowen Li[a,b,1], Boris Podobnik[c,d,e,f], Tobias Preis[c,g,h], and H. Eugene Stanley[c,1]

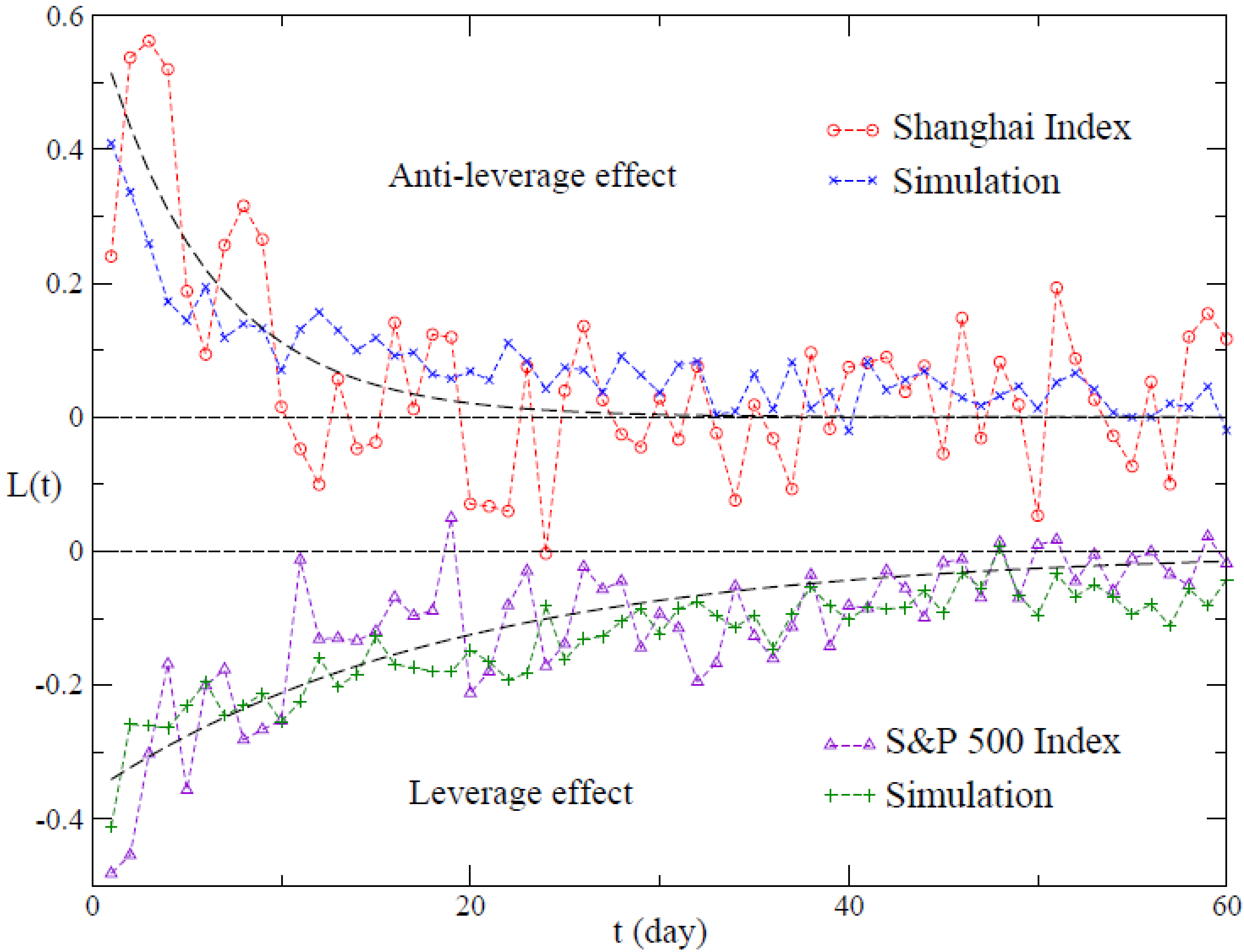
[a]Graduate School for Integrative Sciences and Engineering, National University of Singapore, Singapore 117456, Republic of Singapore; [b]Department of Physics and Centre for Computational Science and Engineering, National University of Singapore, Singapore 117542, Republic of Singapore; [c]Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215; [d]Faculty of Civil Engineering, University of Rijeka, 51000 Rijeka, Croatia; [e]Faculty of Economics, University of Ljubljana, 1000 Ljubljana, Slovenia; [f]Zagreb School of Economics and Management, 10000 Zagreb, Croatia; [g]Chair of Sociology, in particular of Modeling and Simulation, Eidgenössische Technische Hochschule Zurich, 8092 Zurich, Switzerland; and [h]Artemis Capital Asset Management GmbH, 65558 Holzheim, Germany

Contributed by H. Eugene Stanley, March 29, 2012 (sent for review December 6, 2011)

It is well-known that financial asset returns exhibit fat-tailed distributions and long-term memory. These empirical features are the main objectives of modeling efforts using (i) stochastic processes to quantitatively reproduce these features and (ii) agent-based simulations to understand the underlying microscopic interactions. After reviewing selected empirical and theoretical evidence documenting the behavior of traders, we construct an agent-based model to quantitatively demonstrate that "fat" tails

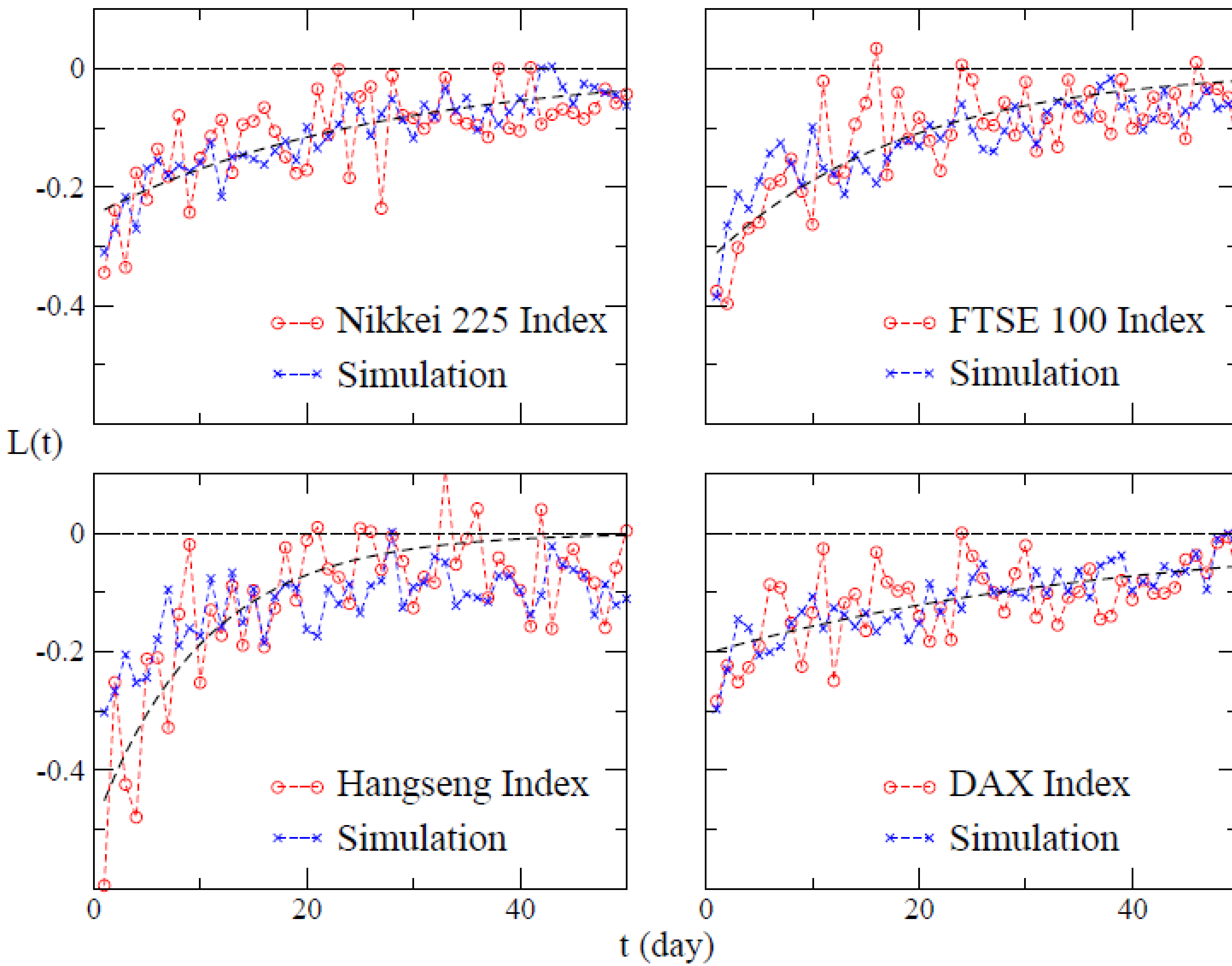
Market surveys (16–18) also provide clear evidence of the prevalence of technical analysis. We consider here only technical traders, assuming that fundamentalists contribute only to market noise. Our study is of the empirical data recorded prior to 2006 and ignores the effect of high frequency trading (HFT) that has become significant only in the past 5 y. We propose a behavioral agent-based model that is in agreement with the following empirical evidence:

Results



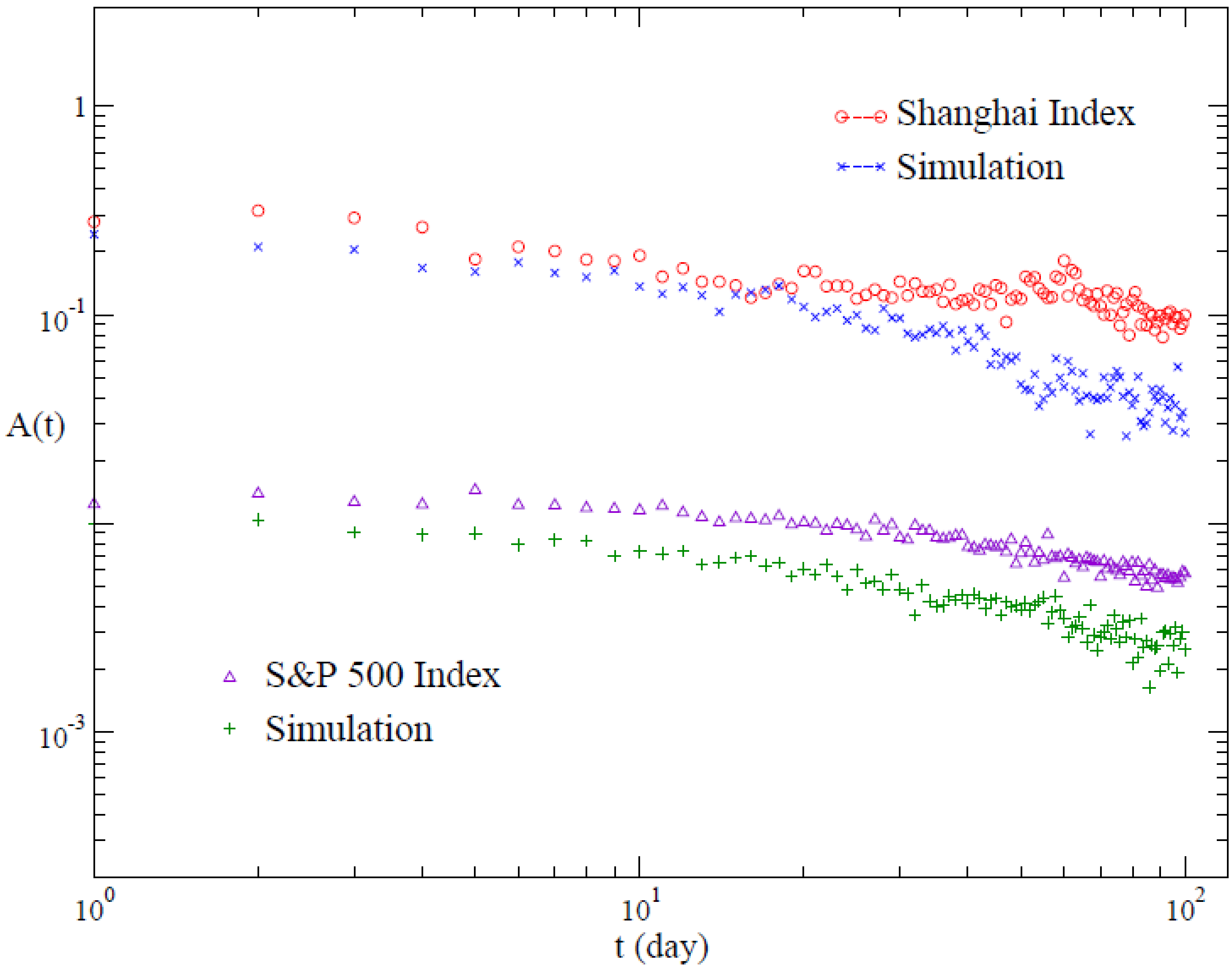
0.6
0.4
0.2
0
-0.2
-0.4
0
L(t)
0
20
40
60
t (day)
Anti-leverage effect
Shanghai Index
Simulation
Leverage effect
S&P 500 Index
Simulation

Results

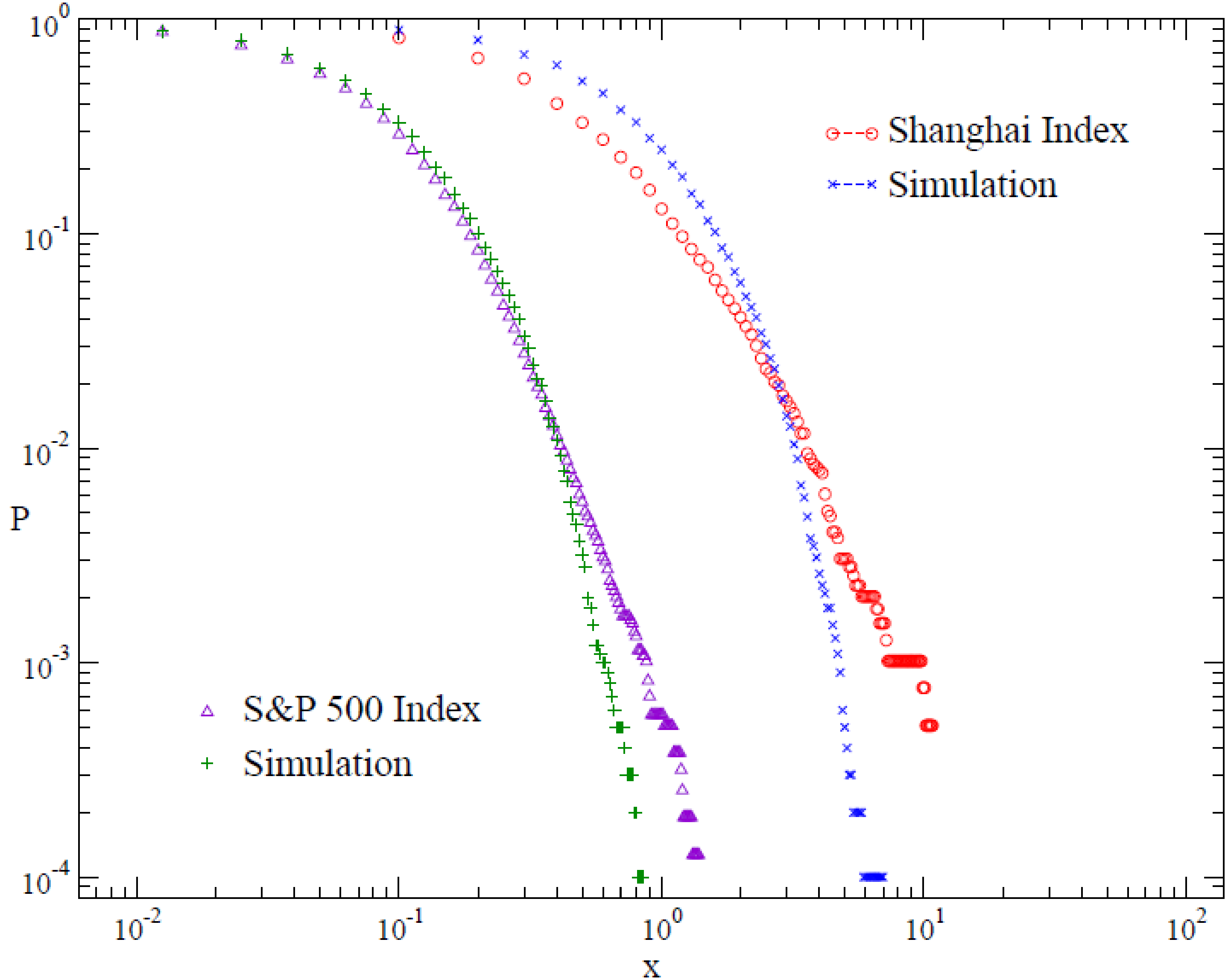


0
-0.2
-0.4
Nikkei 225 Index
Simulation
FTSE 100 Index
Simulation
L(t)
Hangseng Index
Simulation
DAX Index
Simulation
0
20
40
0
20
40
t (day)

Results


Shanghai Index
Simulation
S&P 500 Index
Simulation
A(t)
t (day)
1
10^{-1}
10^{-3}
10^{0}
10^{1}
10^{2}

Results



Shanghai Index
Simulation
S&P 500 Index
Simulation
P
x
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-2}
10^{-1}
10^0
10^1
10^2

The base of our model

- In stock markets, the information for investors is highly incomplete, therefore an agent's decision of *buy*, *sell* or *hold* is assumed to be random.
- Since intraday trading is not persistent in empirical trading data, we consider that only one trading decision is made by each agent in a single day

Agents

In our model, there are *N* agents, and each operates one share every day. On day *t*, each agent *i* makes a trading decision

$$S_i(t) = \begin{cases} 1 & \text{buy} \\ -1 & \text{sell} \\ 0 & \text{hold} \end{cases}$$

the probabilities of buy, sell and hold decisions are denoted as $P_{buy}(t)$, $P_{sell}(t)$ and $P_{hold}(t)$.

Returns

$$S_i(t) = \begin{cases} 1 & \text{buy} \\ -1 & \text{sell} \\ 0 & \text{hold} \end{cases}$$

Return $R(t)$ is defined by the difference of the demand and supply of the stock,

$$R(t) = \sum_{i=1}^{N} S_i(t)$$

Investment horizon

Investors make decisions according to the previous stock performance of different time windows. The relative portion γ_i of agents with *i* days investment horizon follows a power-law decay

$$\gamma_i \propto i^{-\eta} \text{ with } \eta = 1.12 \,.$$

Investment horizon

For an agent having investment horizon of *i* days, $\sum_{j=0}^{i-1} R(t-j)$represents a simplified investment basis for decision making on day $t + 1$.

We introduce a weighted average return $R'(t)$to describe the integrated investment basis of all agents

$$R'(t) = k \cdot \sum_{i=1}^{M} \left[\gamma_i \sum_{j=0}^{i-1} R(t-j) \right]$$

Two investment behaviors

- Investors' asymmetric trading in bull and bear markets: an investor's willingness to trade is distinct in bull and bear markets

- Investors' asymmetric herding in bull and bear markets: investors may cluster more intensively in either bull or bear markets, leading the herding to be asymmetric

Bull and bear markets

Bull and bear markets correspond to the periods of increasing and decreasing stock prices respectively.

Define the previous market to be bullish if $R'(t) > 0$, and bearish if $R'(t) < 0$.

Asymmetric trading

We assume $P_{buy}(t) = P_{sell}(t)$

the trading probability

$P_{trade}(t) = P_{buy}(t) + P_{sell}(t)$

Since $N \cdot P_{trade}(t)$ is the trading volume on day *t*, we set its average over time

$\langle P_{trade}(t) \rangle = 2p$, $p = 0.0154$

Asymmetric trading

The asymmetric trading gives rise to the distinction between $P_{trade}(t+1)|_{R'(t)>0}$ and $P_{trade}(t+1)|_{R'(t)<0}$. Therefore, $P_{trade}(t+1)$ should take the form

$$\begin{cases} P_{trade}(t+1) = 2p \cdot \alpha & R'(t) > 0 \\ P_{trade}(t+1) = 2p & R'(t) = 0 \\ P_{trade}(t+1) = 2p \cdot \beta & R'(t) < 0 \end{cases}$$

$\langle P_{trade}(t) \rangle = 2p$ requires $\alpha + \beta = 2$.

Asymmetric herding

$D(t)$ is introduced to quantify the clustering degree of the herding behavior

$$D(t) = n_A(t)/N$$

Herding should be related to previous volatilities, we set $n_A(t+1) = |R'(t)|$ thus

$$D(t+1) = |R'(t)|/N$$

According to the asymmetric herding

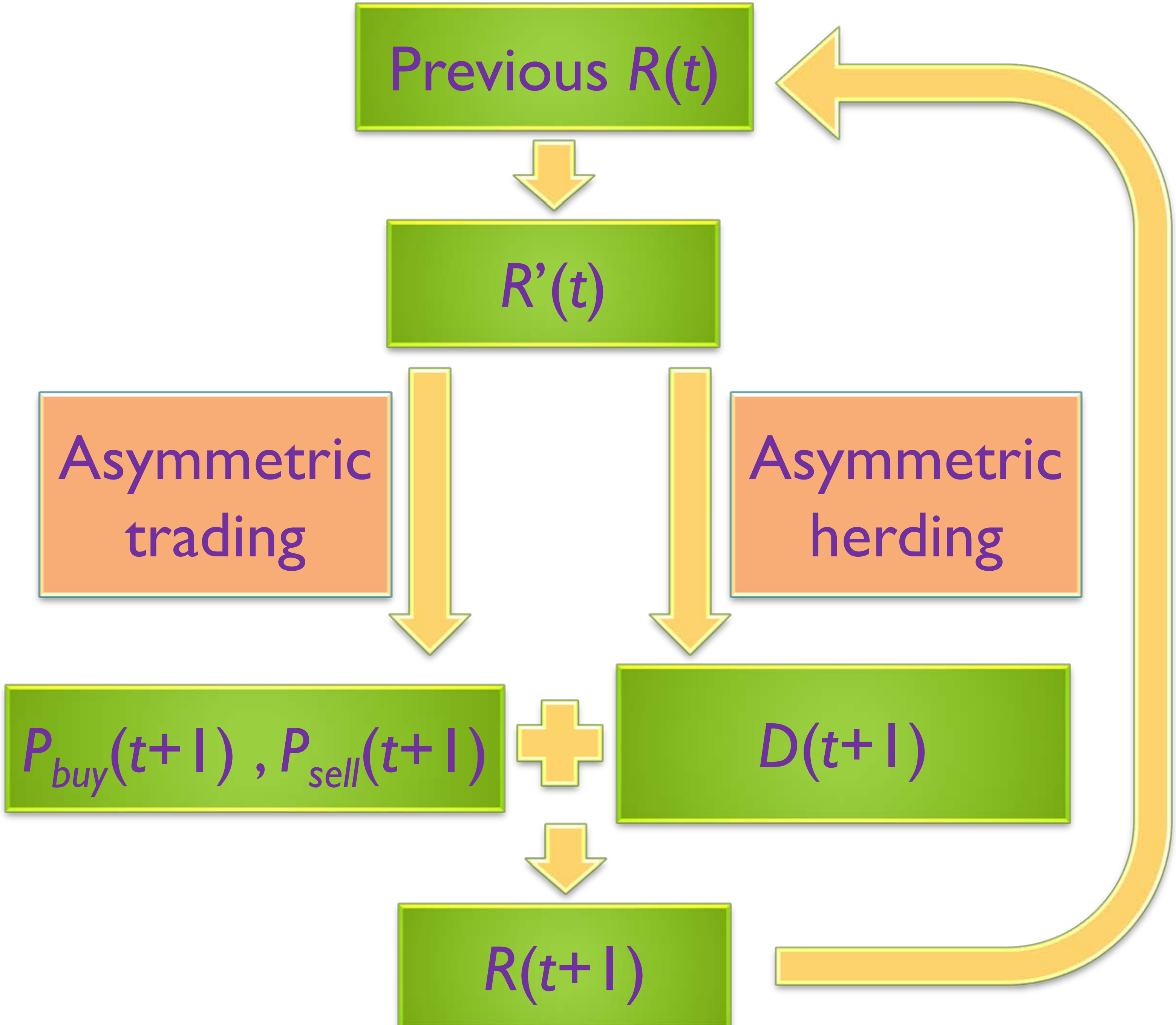
$$D(t+1) = |R'(t) - \Delta R|/N$$

Asymmetric herding

$$D(t+1) = n_A(t+1)/N$$

$N \cdot D(t+1)$ is the average number of agents in a same group. Thus on day t+1, we randomly divide all agents into $1/D(t+1)$ groups.

Everyday, the agents in a group make a same trading decision (buy, sell or hold) with the same probability


Previous $R(t)$
$R'(t)$
Asymmetric trading
Asymmetric herding
$P_{buy}(t+1)$, $P_{sell}(t+1)$
$D(t+1)$
$R(t+1)$

Key Parameters

$$\begin{cases} P_{trade}(t+1) = 2p \cdot \alpha & R'(t) > 0 \\ P_{trade}(t+1) = 2p & R'(t) = 0 \\ P_{trade}(t+1) = 2p \cdot \beta & R'(t) < 0 \end{cases}$$

$$\alpha + \beta = 2$$

$$D(t+1) = |R'(t) - \Delta R|/N$$

The parameters are determined from historical market data rather than from statistical fitting of the simulated results.

Determination of α

The stock market is assumed to be bullish if $r(t) > 0$, and bearish if $r(t) < 0$. We define

$$\begin{cases} V_+ = [\sum_{r(t)>0} V(t)]/n_{r(t)>0} \\ V_- = [\sum_{r(t)<0} V(t)]/n_{r(t)<0} \end{cases}$$

In our model, the average trading volume is $N \cdot P_{trade}(t+1)|_{R'(t)>0}$ for bull markets, and $N \cdot P_{trade}(t+1)|_{R'(t)<0}$ for bear ones.

$$\frac{P_{trade}(t+1)|_{R'(t)>0}}{P_{trade}(t+1)|_{R'(t)<0}} = \alpha/\beta = V_+/V_-$$

Determination of ΔR

$$d_{bull}(r(t)) = \sum_{t,r(t)>0}[V(t)\cdot r(t)]/\sum_{t,r(t)>0}V(t)$$
$$d_{bear}(r(t)) = \sum_{t,r(t)<0}[V(t)\cdot |r(t)|]/\sum_{t,r(t)<0}V(t)$$

We introduce a shifting to *r(t)*, such that

$$d_{bull}(r'(t)) = d_{bear}(r'(t)) \text{ with } r'(t) = r(t) + \Delta r$$

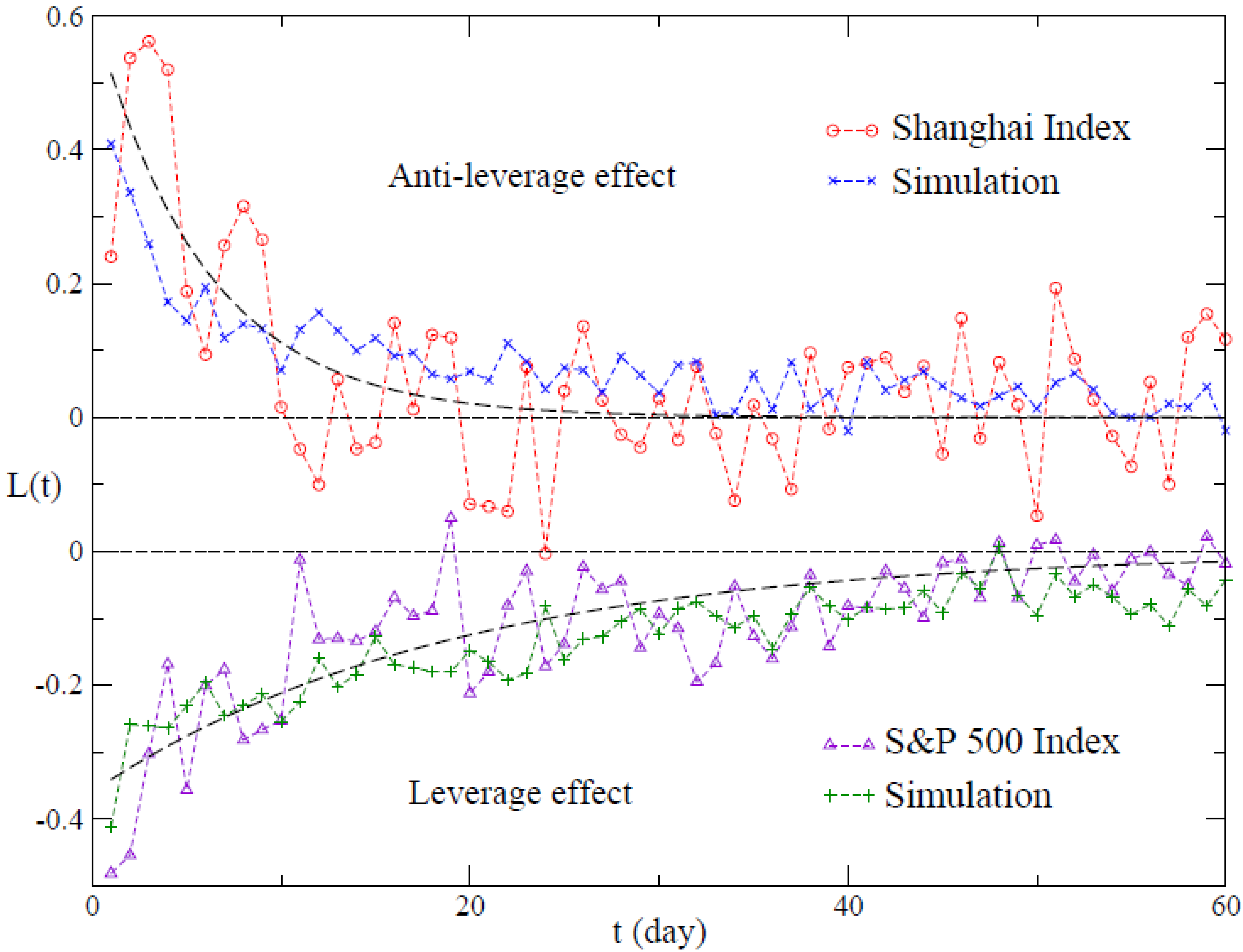
$$\Delta r = \frac{1}{2}\left\{\frac{\sum_{t,r(t)<0}[V(t)\cdot|r(t)|]}{\sum_{t,r(t)<0}V(t)} - \frac{\sum_{t,r(t)>0}[V(t)\cdot r(t)]}{\sum_{t,r(t)>0}V(t)}\right\}$$

$$[\Delta R - \langle R(t)\rangle]/\sigma = \Delta r$$

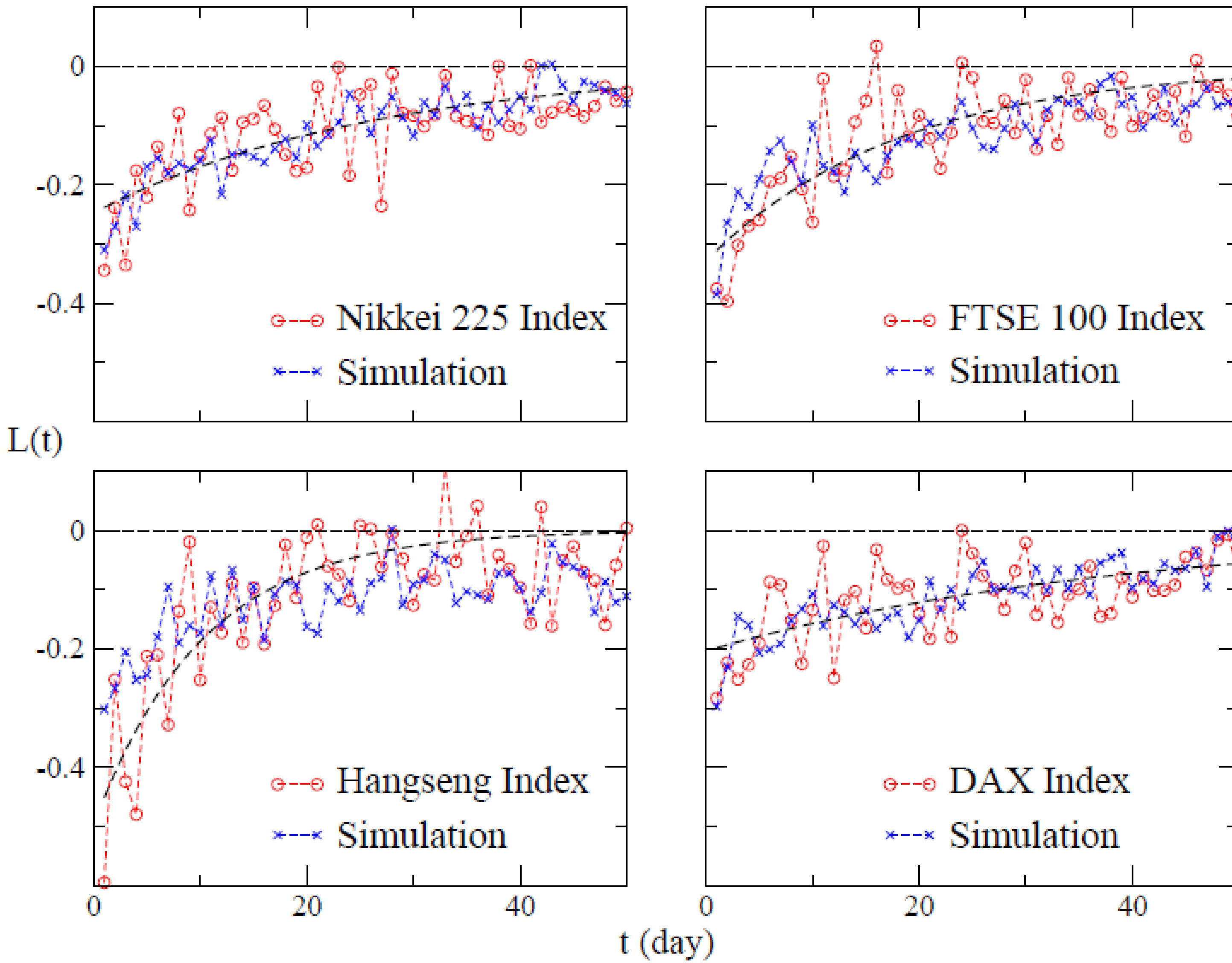
Results

Index	V_+/V_-	α	β	Δr	ΔR
S&P 500 (1950-2012)	1.03	1.01	0.99	0.067	3
Shanghai (1991-2006)	1.21	1.09	0.91	-0.043	-2
Nikkei 225 (2003-2012)	1.01	1.01	0.99	0.039	2
FTSE 100 (2004-2012)	0.98	0.99	1.01	0.028	2
Hangseng (2001-2012)	1.04	1.02	0.98	0.032	2
DAX (2008-2012)	0.96	0.98	1.02	0.013	1

Results


Shanghai Index
Simulation
Anti-leverage effect
S&P 500 Index
Simulation
Leverage effect
L(t)
t (day)
0.6
0.4
0.2
0
0
-0.2
-0.4
0
20
40
60

Results



0
-0.2
-0.4
Nikkei 225 Index
Simulation
FTSE 100 Index
Simulation
L(t)
Hangseng Index
Simulation
DAX Index
Simulation
0
20
40
t (day)

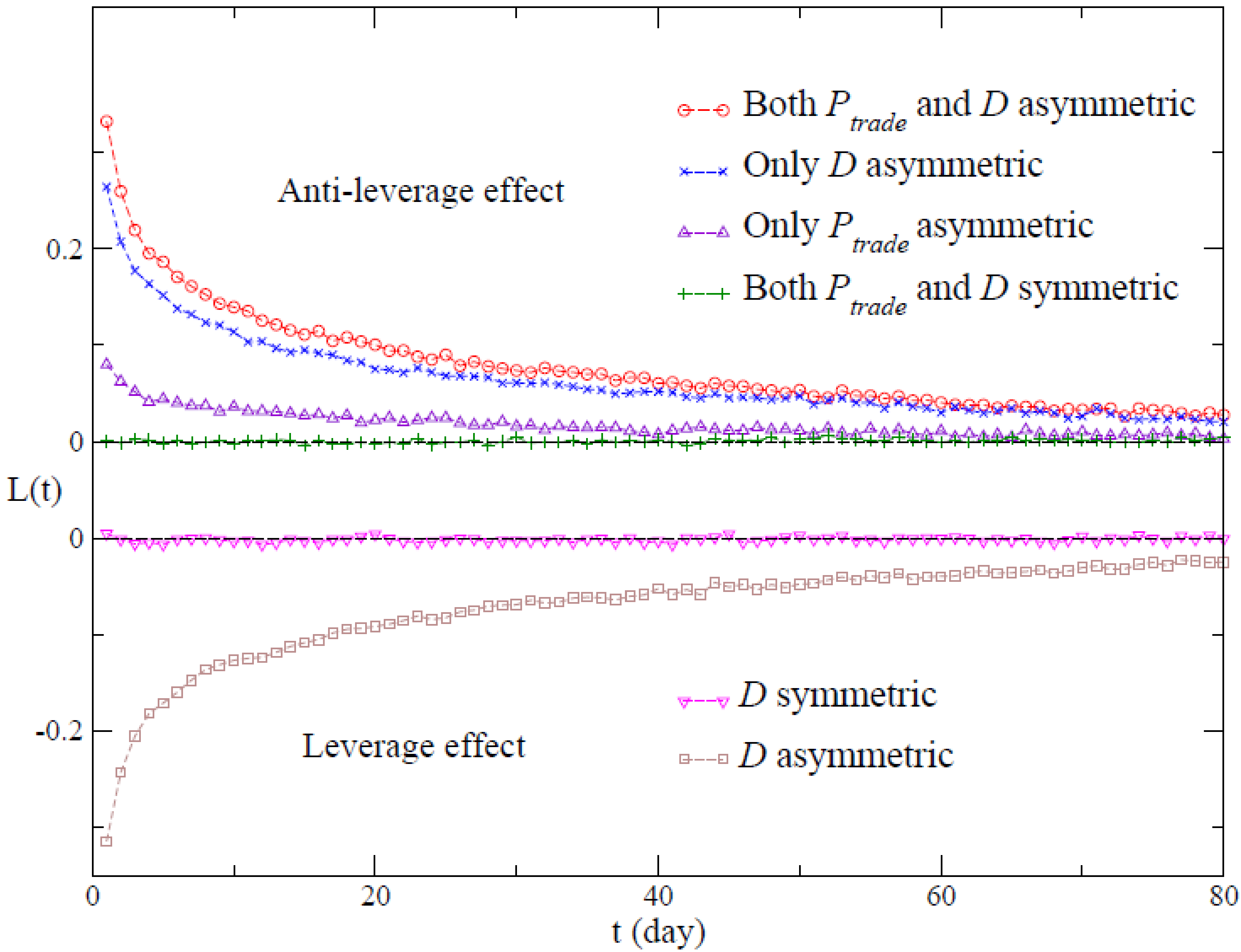
Discussion

P_{trade} is symmetric in the simulation of the S&P 500 Index, but asymmetric in the simulation of the Shanghai Index.

$D(t+1)$ is asymmetric in the simulation of both the S&P 500 and Shanghai indices.

With other parts of the model remain unchanged, we consider the following controls :

Results



Both P_{trade} and D asymmetric
Only D asymmetric
Only P_{trade} asymmetric
Both P_{trade} and D symmetric
Anti-leverage effect
0.2
0
L(t)
0
-0.2
D symmetric
D asymmetric
Leverage effect
0
20
40
60
80
t (day)

Conclusions

- We construct an agent-based model with asymmetric trading and herding.
- With α and ΔR determined for six indices respectively, we obtain the corresponding leverage or anti-leverage effect from the simulation.
- For the leverage and anti-leverage effects, both the asymmetric trading and herding are essential generation mechanisms.

Thank you!