Magnetic domain-wall motion and dynamic phase transitions in low dimensional materials

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Publications

93. Qin, **B. Zheng**, and N.J. Zhou, Depinning phase transition in the two-dimensional clock model with quenched randomness, **Phys. Rev. E 86** (2012) 031129
92. R.H. Dong, **B. Zheng**, and N.J. Zhou, Hamiltonian equation of motion and depinning phase transition in two-dimensional magnets, **EPL 99** (2012) 56001
91. R.H. Dong, **B. Zheng**, and N.J. Zhou, Creep motion of a domain wall in the two-dimensional random-field Ising model with a driving field, **EPL 98** (2012) 36002
89. N.J. Zhou, **B. Zheng** and D.P. Landau, Modeling relaxation-to-creep transition of domain-wall motion in ultrathin ferromagnetic films, **EPL 92** (2010) 36001
88. N.J. Zhou and **B. Zheng,** Dynamic effect of overhangs and islands at the depinning transition in two-dimensional magnets, **Phys. Rev E82** (2010) 031139
84. N.J. Zhou, **B. Zheng**, and Y.Y. He, Short-time domain-wall dynamics in the random-field Ising model with a driving field, **Phys. Rev. B80** (2009) 134425

Outline

I Introduction

II Depinning phase transition

III Relaxation-to-creep transition

Domain-wall dynamics in magnetic devices, nano-materials, and semi-conductors etc

Science 317, 1726 (2007)
320, 190 (2008)
Phys. Rev. Lett. 108, 247202 (2012)
109, 167209 (2012)
106, 087204 (2011)
102, 045701 (2009)
101, 207203 (2008)
98, 255502 (2007)
Phys. Rev. B 80, 214426 (2009)
80, 052409 (2009)
78, 161303 (2008)

It plays a key role in field-induced and Current-induced magnetization reversal

Domain-wall motion in ultra-thin magnetic films

Phys. Rev. Lett. 80, 849 (1998)
Nattermann et al
Phys. Rev. Lett. 90, 047201 (2003)
87, 197005 (2001)
Phys. Rev. B 59, 4260 (1999)
Kleemann et al
Phys. Rev. Lett. 99, 097203 (2007)
97, 065702 (2006)
94, 117601 (2005)
89, 137203 (2002)
Phys. Rev. B 70, 134108 (2004)
70, 214432 (2004)

Quenched disorder and external magnetic field

Experiments of ultrathin magnetic films

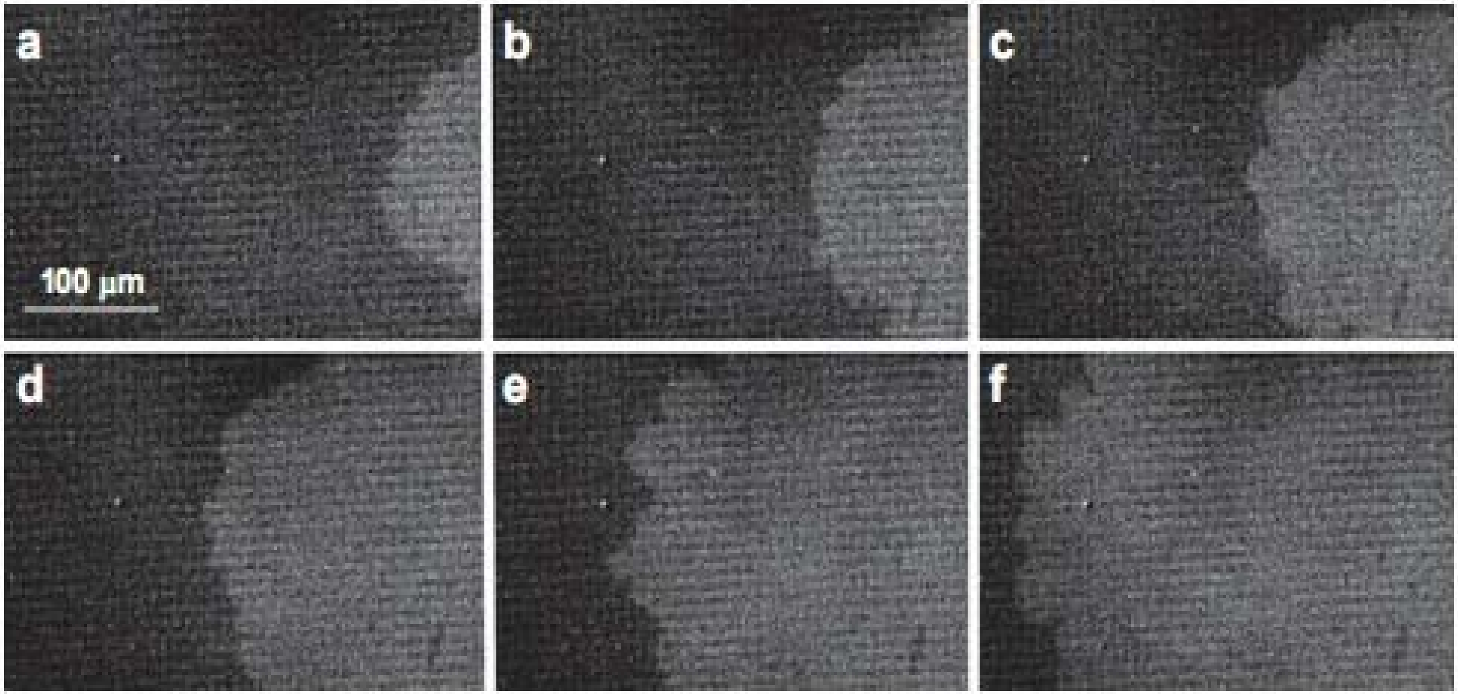


Figure 15

Domain wall in Pt(0.35 nm)/Co(0.5 nm)/Pt(0.45 nm) driven at room temperature by a perpendicular magnetic field $\mu_0 H_0 = 0.42$ mT shown at successive time intervals (*a*–*f*) of $\Delta t = 0.5$ s. From W. Kleemann, J. Rhensius, O. Petracic, J. Ferré, J.P. Jamet, H. Bernas (unpublished data).

Experiment of magnetic domain wall

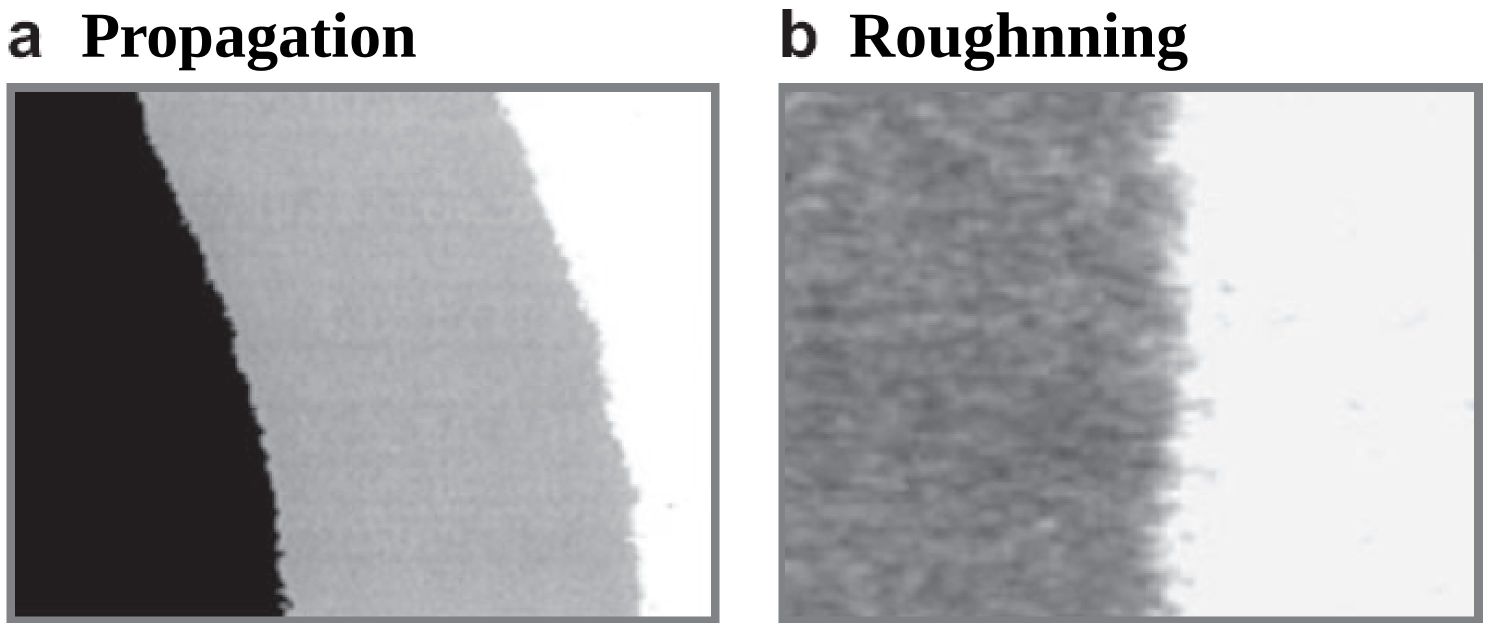
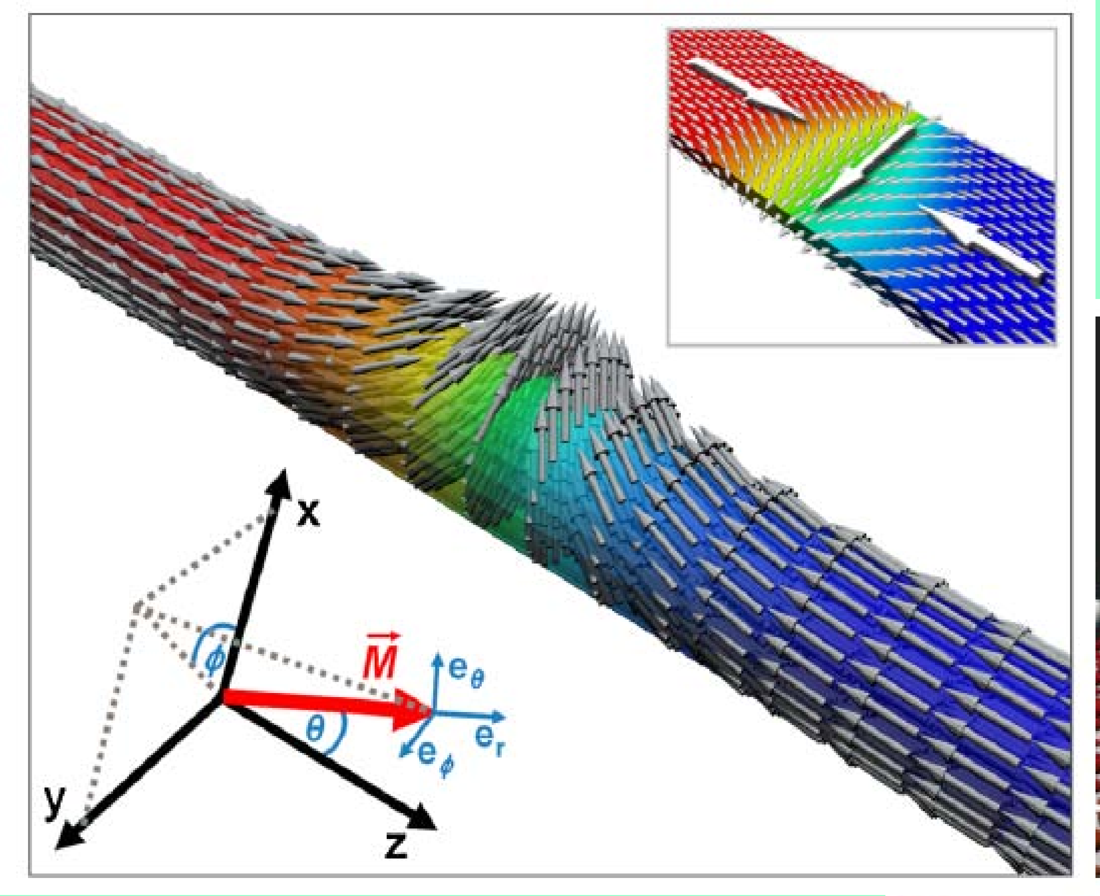
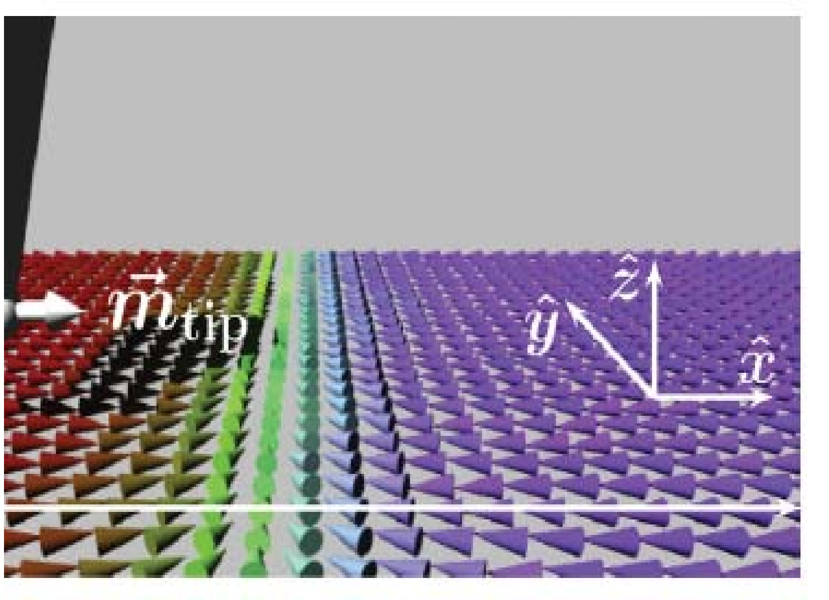


Figure 1

(*a*) Polar Kerr magneto-optic image (size 90 × 72 μm^2) of a domain wall (DW) in a multilayer Si/Si_3N_4/Pt(6.5 nm)/Co(0.5 nm)/Pt(3.4 nm) with perpendicular magnetic anisotropy before (*black*) and after (*gray*) being swept by a perpendicular field of 46 mT during 111 μs. From Reference 9. Reproduced by permission of the American Physical Society. (*b*) Rough DW in a thick film of $PbZr_{0.2}Ti_{0.8}O_3$ (PZT) (size 500 × 500 μm^2, thickness 66 nm) recorded by scanning piezoforce response microscopy, revealing attractive and repulsive forces at up and down polarized domains (*dark* and *light gray*, respectively). From Reference 10. Reproduced by permission of the American Physical Society.

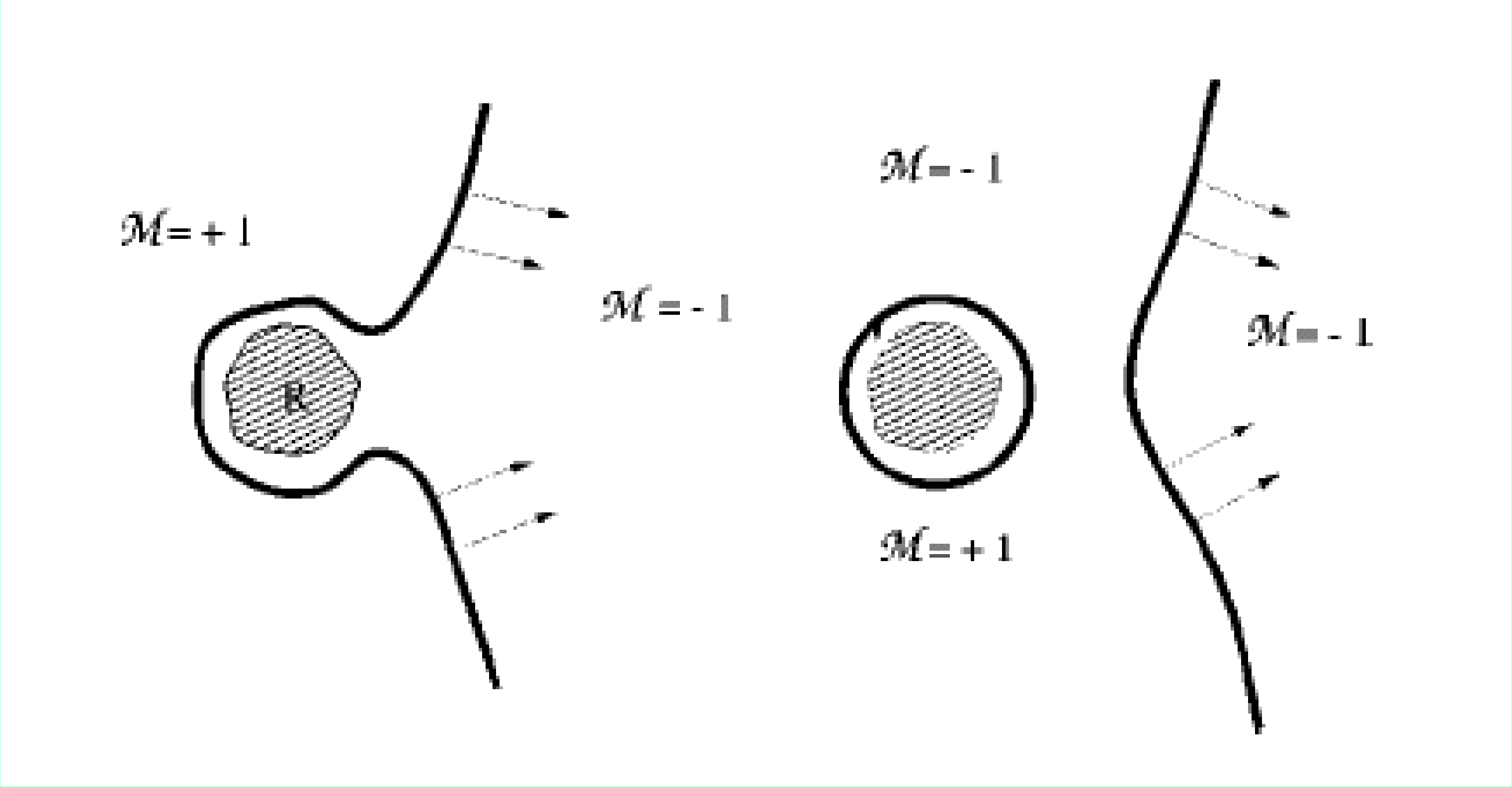
Schematic domain walls
in magnetic nano-wire, strip, film



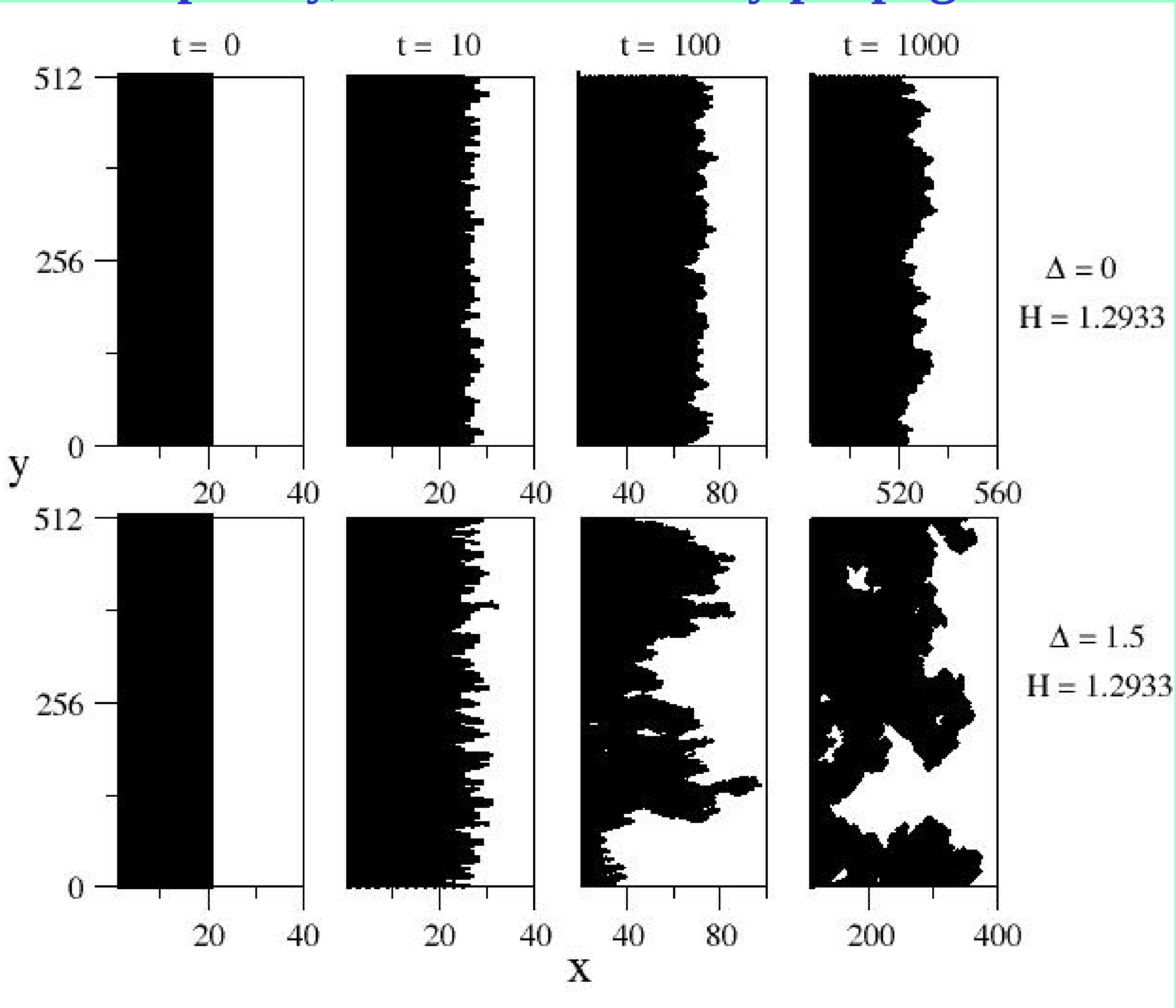


Microscopically,

Domain wall may move and create overhangs and islands



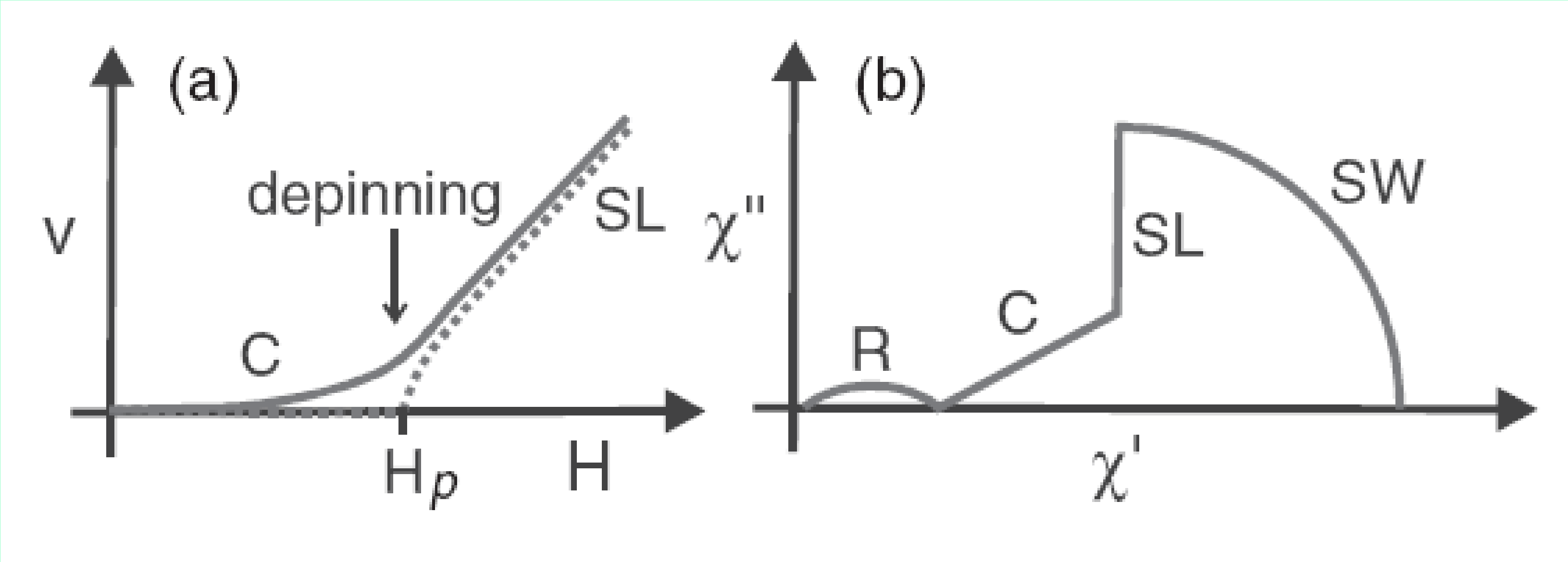
Macroscopically, domain wall may propagate and roughen



T = 0

H = Hp

Random field Ising model



Four states of domain walls

*** Relaxation, creep, slide, switch**

Dynamic phase transitions

*** Relaxation-to-creep**
*** Creep-slide**
(at zero temperature, depinning transition)

Theoretical approaches

I. Quenched Edwards-Wilkinson equation (QEW)

simple and phenomenological

$$\frac{1}{\gamma}\frac{\partial z}{\partial t} = \Gamma \Delta z + h_0 \cos \omega_0 t + g(\vec{x}, z)$$

II. Lattice models + Monte Carlo simulations

more realistic, but dynamics is approximate

III. Molecular dynamics simulations

more fundamental

- **Landau-Lifshitz-Gilbert equation (LLG)**
- ϕ^4 **theory**

Our Motivation

- **Microscopic lattice models**
- **Monte Carlo simulations and molecular dynamics simulations**
- **Depinning transition and relaxation-to-creep transition**
- **Non-steady state**

Outline

I Introduction

II Depinning phase transition

III Relaxation-to-creep transition

Pinning-depinning phase transition

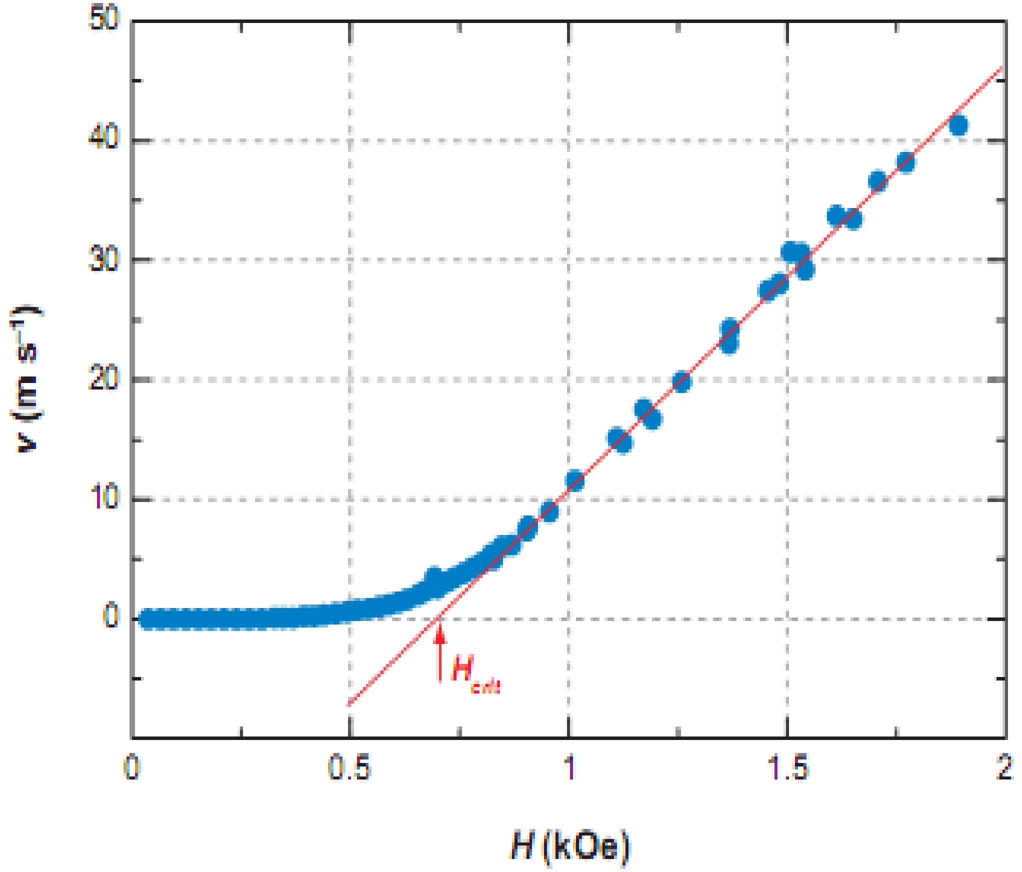


Figure 2

Magnetic domain wall velocity in Pt(6.5 nm)/Co(0.5 nm)/Pt(3.4 nm) versus applied magnetic field at room temperature (in m s^{-1}). The red line is the linear fit of the high field part, $H > 0.86$ kOe. The arrow marks its intersection with the line $v(H) = 0$, which defines the depinning field $H_{crit} \equiv H_p$. From Reference 9. Reproduced by permission of the American Physical Society.

Random-field Ising model

Phys. Rev. B80 (2009) 134425

$$\mathcal{H} = -J\sum_{\langle ij\rangle} S_i S_j - \sum_i (h_i + H) S_i .$$

Initial state:

a perfect domain wall with *v=0*

Dynamics:

Monte Carlo simulation at *T=0*

Typical method:

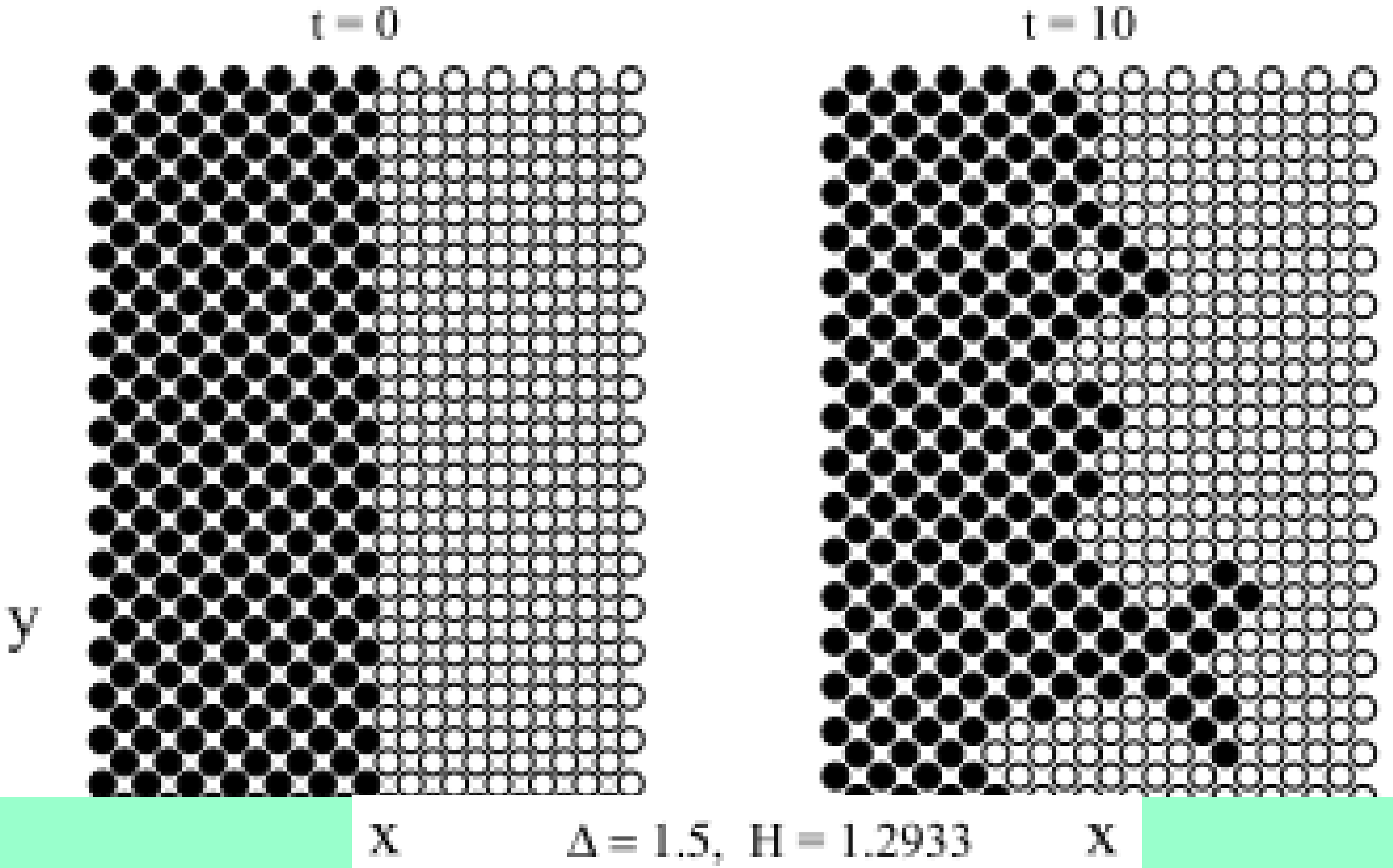
Finite-size scaling in steady state

Our approach:

Dynamic scaling form in non-steady state (i.e., Short-time dynamic scaling form)

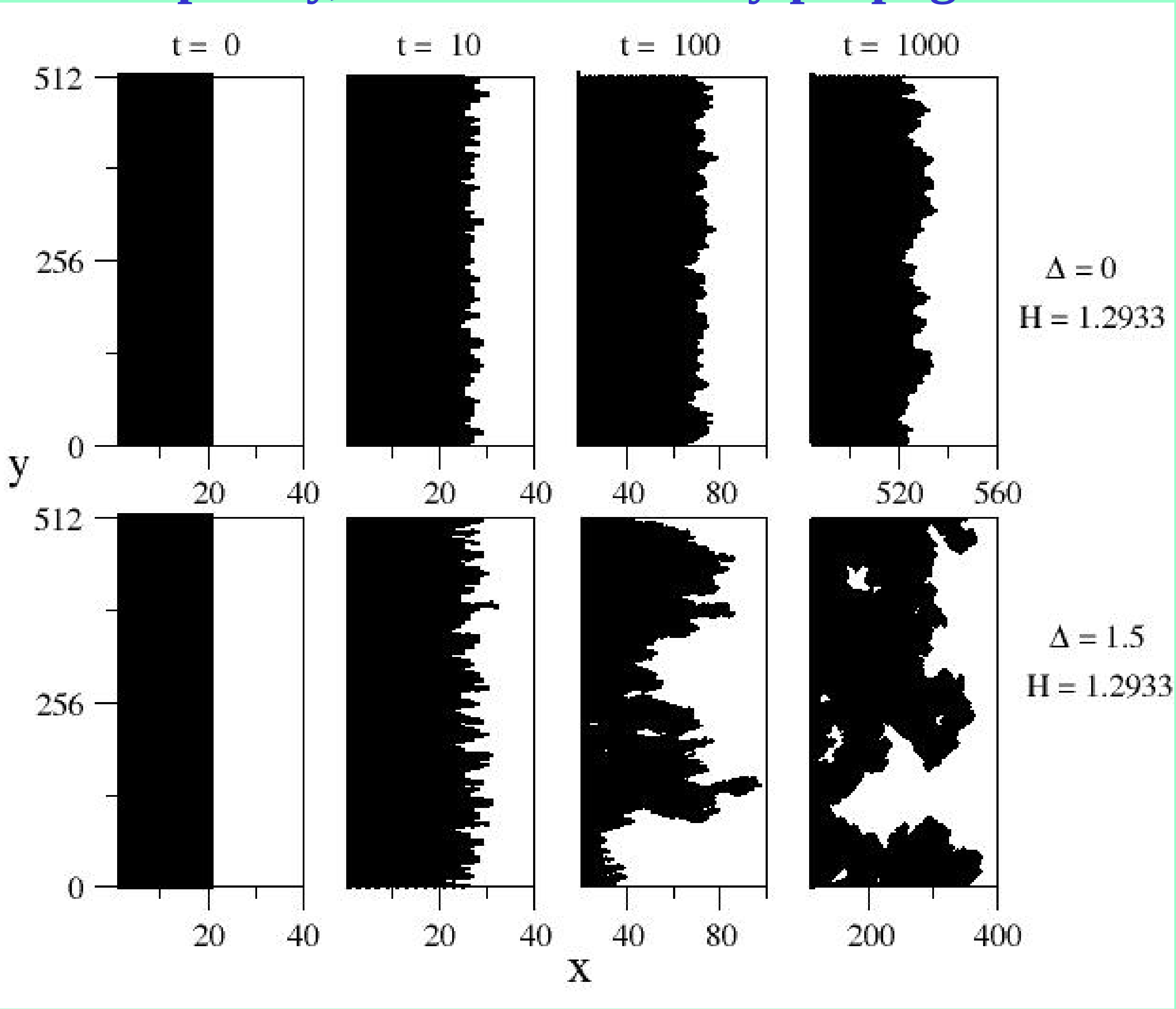
E.g., at *Tc*, how the velocity relaxes to zero

The measurment does not suffer from critical slowing down



$\Delta = 1.5,\ H = 1.2933$

Macroscopically, domain wall may propagate and roughen



T = 0

H = Hp

Random field Ising model

- **Height function is not unique, e.g.,**

$$h_m(y,t) = \frac{L}{2}[m(y,t)+1].$$

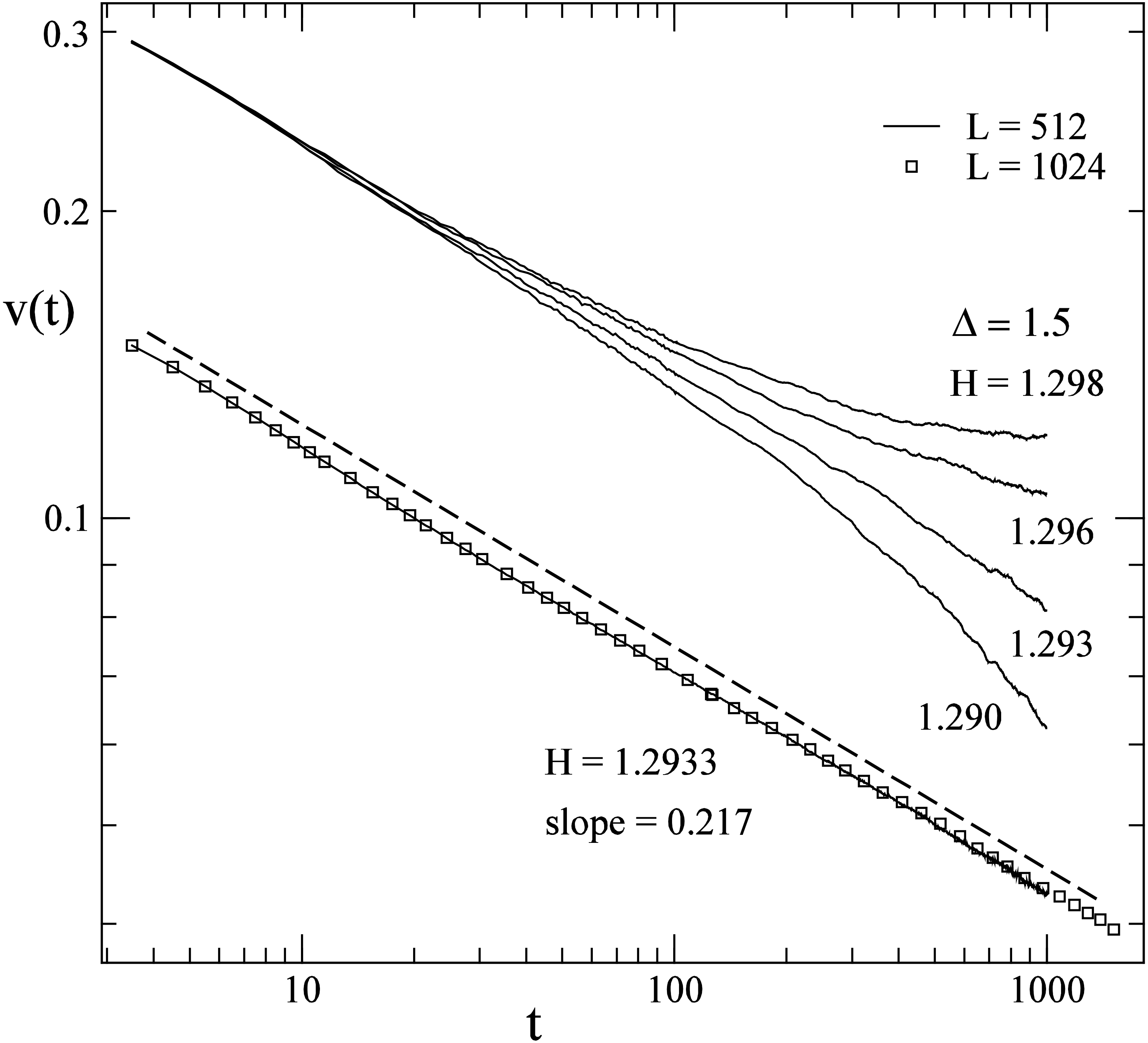
- **Average velocity of the domain wall**

$$v(t) = \frac{L}{2}\frac{dh(t)}{dt}.$$

- **The scaling form of the velocity**

$$v(t,\tau,L) = t^{-\beta/\nu z} v(1, t^{1/\nu z}\tau, t^{-1/z}L).$$

At transition point $v(t) \sim t^{-\beta/\nu z}.$


0.3
0.2
0.1
v(t)
L = 512
L = 1024
Δ = 1.5
H = 1.298
1.296
1.293
1.290
H = 1.2933
slope = 0.217
10
100
1000
t

- **Roughness function**

$$\omega^2(t) = h^{(2)}(t) - h(t)h(t)$$

$$\propto \; t^{2\varsigma/z}$$

- **Fluctuation ratio**

$$F(t) = \frac{M^{(2)}(t) - M(t)M(t)}{h^{(2)}(t) - h(t)h(t)}$$

$$\propto \; \xi(t) \quad \propto \; t^{1/z}$$

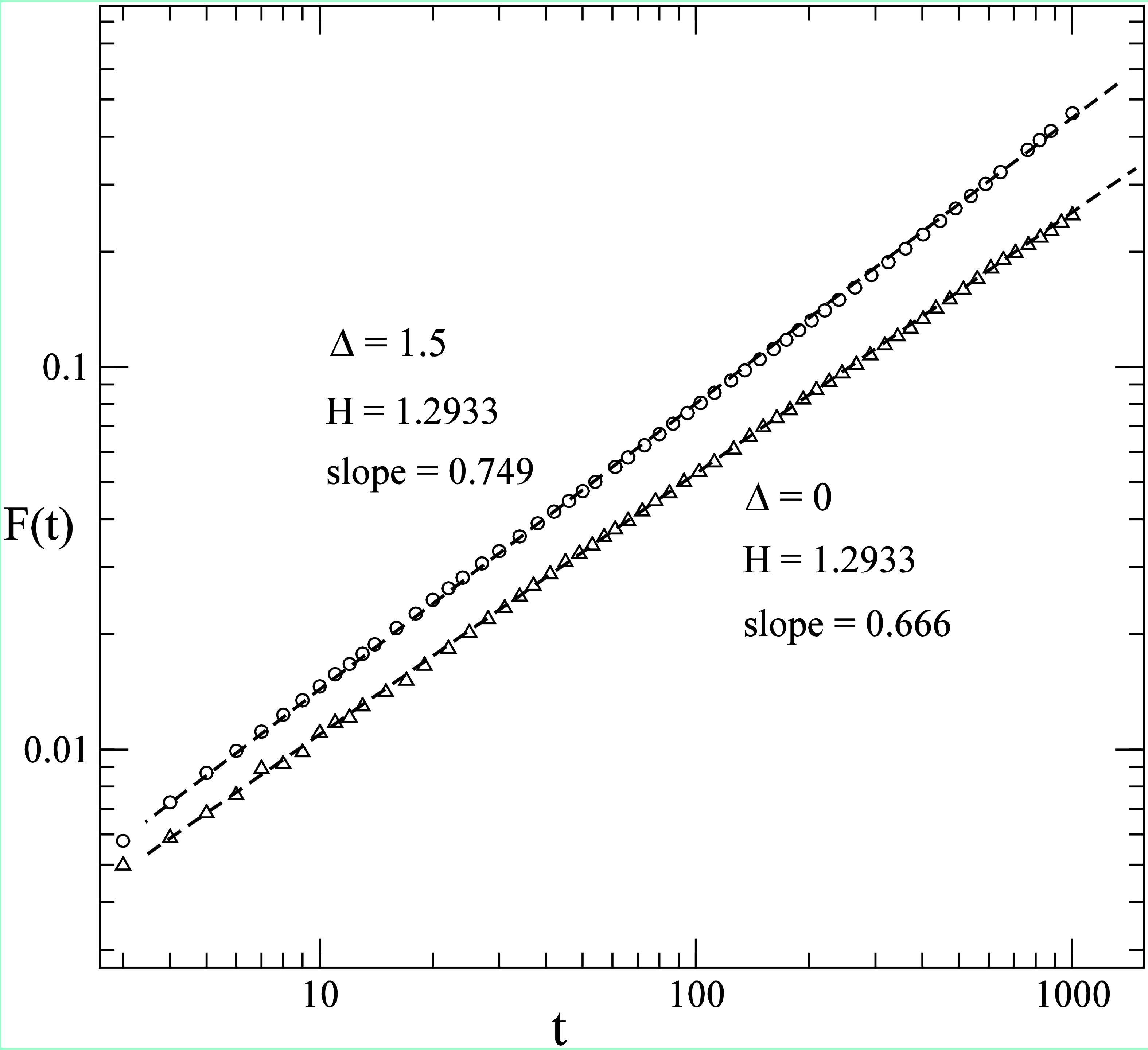
- **Height correlation function**

$$C(r,t) = \langle [h(y+r,t) - h(y,t)]^2 \rangle$$

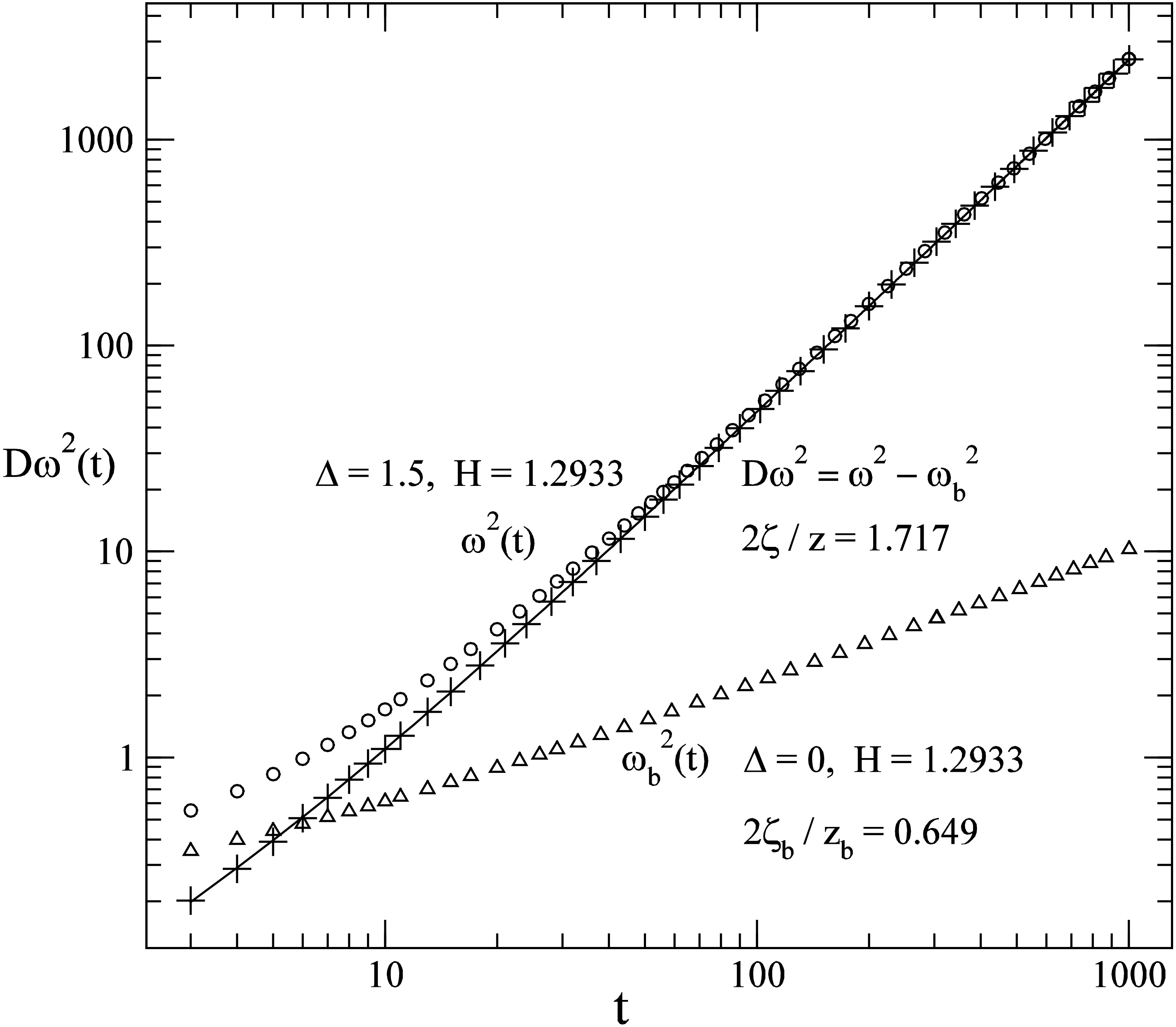
$$\propto \begin{cases} \xi(t)^{2(\zeta-\zeta_{loc})} r^{2\zeta_{loc}} & \text{if } r \ll \xi(t) \ll L \\ \xi(t)^{2\zeta} & \text{if } 0 \ll \xi(t) \ll r \end{cases} .$$

- **Logarithmic derivative**

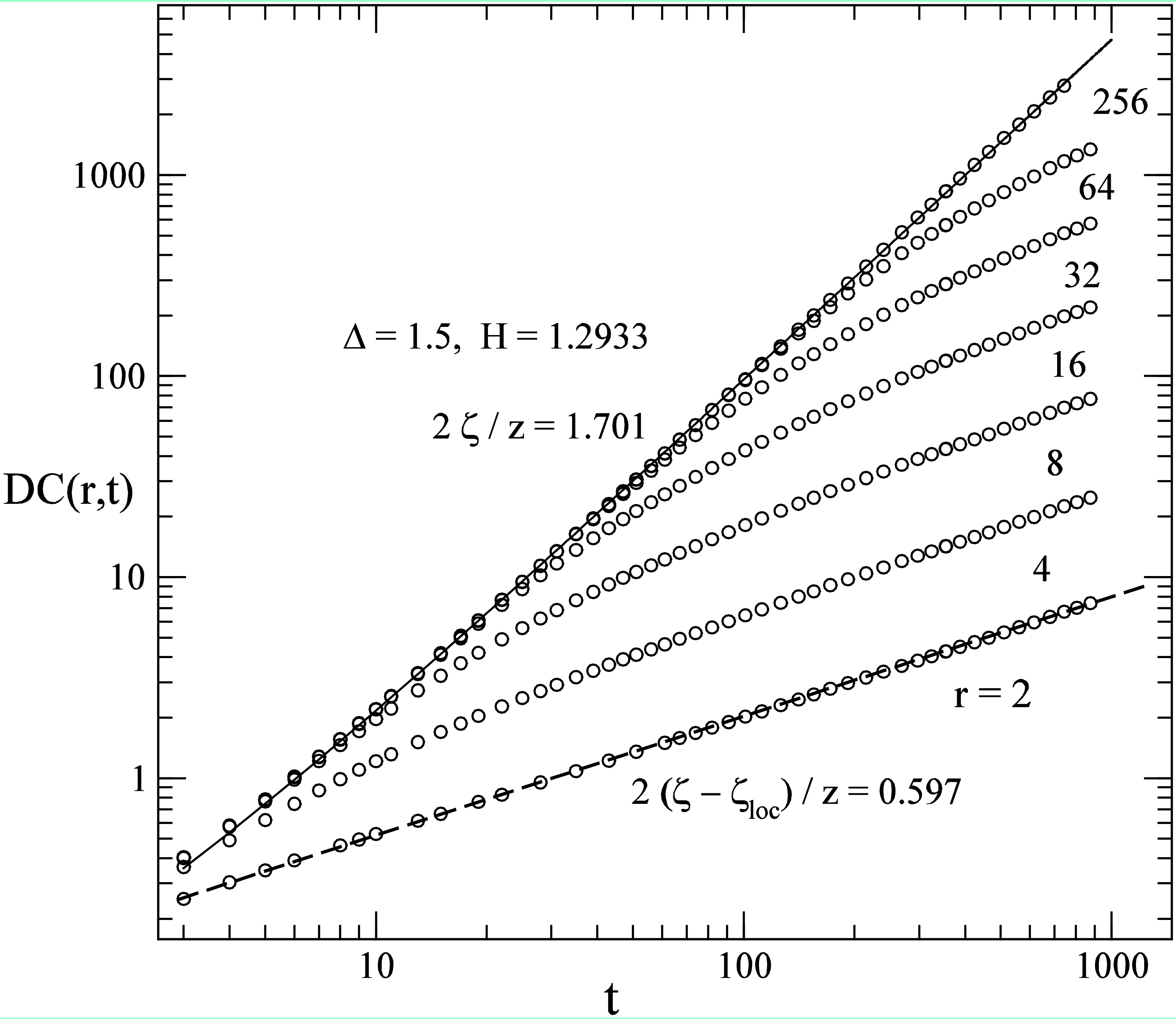
$$\partial_\tau \ln v(t,\tau)|_{\tau=0} \sim t^{1/\nu z} .$$



Δ = 1.5
H = 1.2933
slope = 0.749
Δ = 0
H = 1.2933
slope = 0.666
F(t)
0.1
0.01
10
100
1000
t



$D\omega^2(t)$
$\Delta = 1.5,\ H = 1.2933$
$\omega^2(t)$
$D\omega^2 = \omega^2 - \omega_b^{\ 2}$
$2\zeta / z = 1.717$
$\omega_b^{\ 2}(t)$
$\Delta = 0,\ H = 1.2933$
$2\zeta_b / z_b = 0.649$
1000
100
10
1
10
100
1000
t



Δ = 1.5, H = 1.2933
2 ζ / z = 1.701
2 (ζ − ζloc) / z = 0.597
DC(r,t)
t
256
64
32
16
8
4
r = 2
1000
100
10
1
10
100
1000

		QEW	Magnetization
$v(t)$	H_c		1.2933(2)
	β	0.33(2); 0.33	0.295(3)
	ν	1.29(5); 1.33; 1.33(1)	1.02(2)
	z	1.5; 1.53	1.33(1)
	θ		
$\omega^2(t)$	ζ	1.26(1); 1.25; 1.24	1.14(1)
$C(r,t)$	ζ	1.23(1); 1.25	1.13(1)
	ζ_{loc}	0.98; 0.92	0.735(8)

Dependence on the strength of disorder

* weak universality

* close to QEW equation for weaker disorder except for the exponent ν , but crossover to a first order transition

Overhang and island play a crucial role

They are also critical

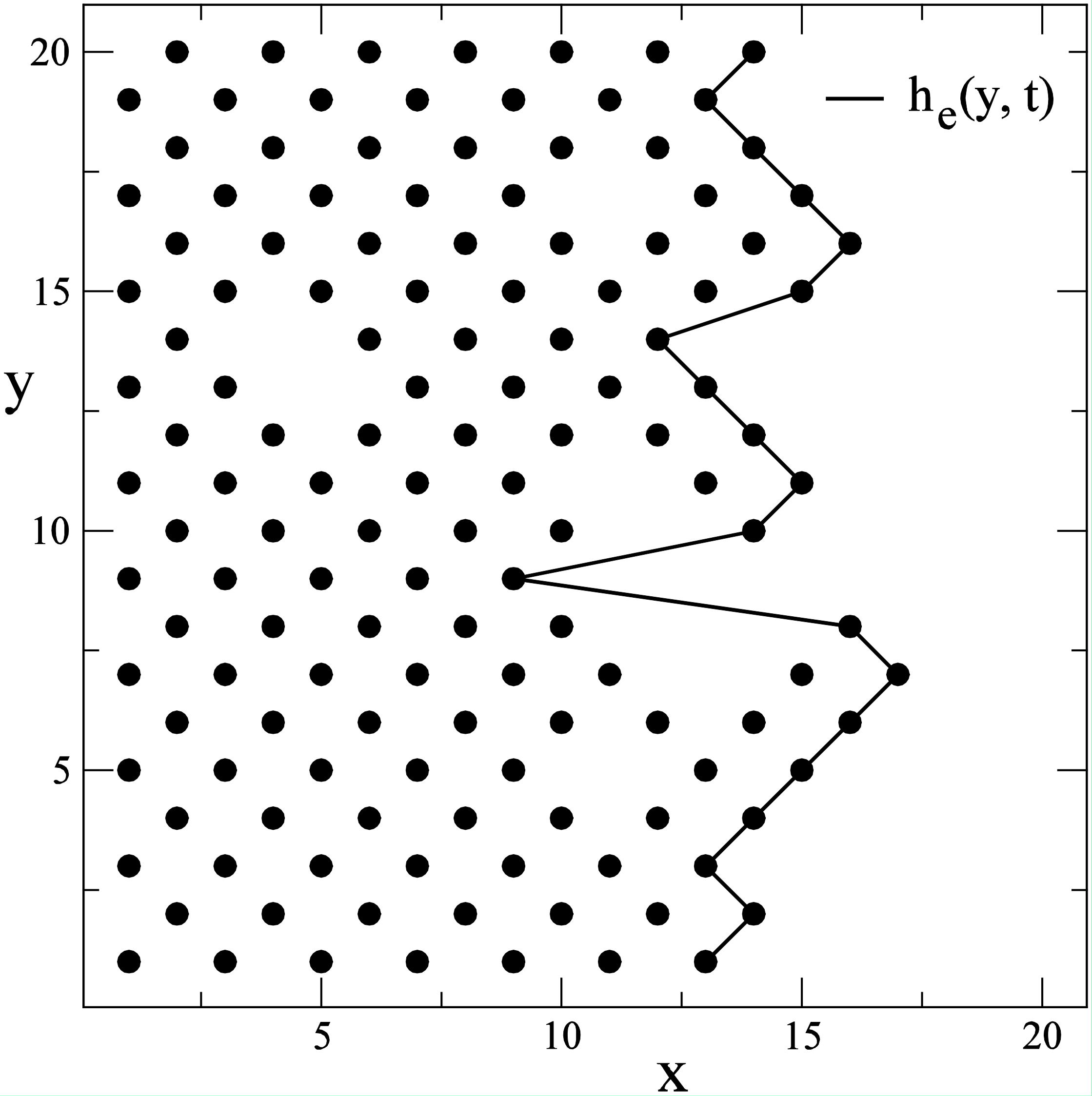
Phys. Rev E82 (2010) 031139

	DRFIM						
		Δ=1.3	Δ=1.5	Δ=1.7	Δ=2.0	Δ=2.3	$\Delta \to \infty$
$v(t)$	H_c	1.2028(2)	1.2933(2)	1.3670(2)	1.4599(2)	1.5398(2)	1.71
	β	0.349(3)	0.295(3)	0.296(3)	0.295(3)	0.298(3)	0.296
	ν	0.99(1)	1.02(2)	1.10(1)	1.17(1)	1.20(2)	1.23
	z	1.43(1)	1.33(1)	1.30(1)	1.27(1)	1.26(1)	1.25
$\omega^2(t)$	ζ	1.20(1)	1.14(1)	1.10(1)	1.06(1)	1.04(1)	1.01
$C(r,t)$	ζ	1.20(1)	1.13(1)	1.10(1)	1.06(1)	1.04(1)	
	ζ_{loc}	0.786(5)	0.735(8)	0.694(7)	0.645(6)	0.608(8)	0.50
	z	1.42(1)	1.33(1)	1.29(1)	1.28(1)	1.26(1)	
	$\frac{1}{2}(\zeta + \zeta_{loc})$	0.99(1)	0.93(1)	0.90(1)	0.85(1)	0.82(1)	0.76
$S(k,t)$	ζ_s	1.01(1)	0.94(1)	0.91(1)	0.84(1)	0.80(1)	0.75

LE II: The depinning transition field and critical exponents obtained for DRFIM for diff

om field Δ. The H_c and the critical exponents for $\Delta \to \infty$ can be obtained with extrapola

Height function defined with the envelop



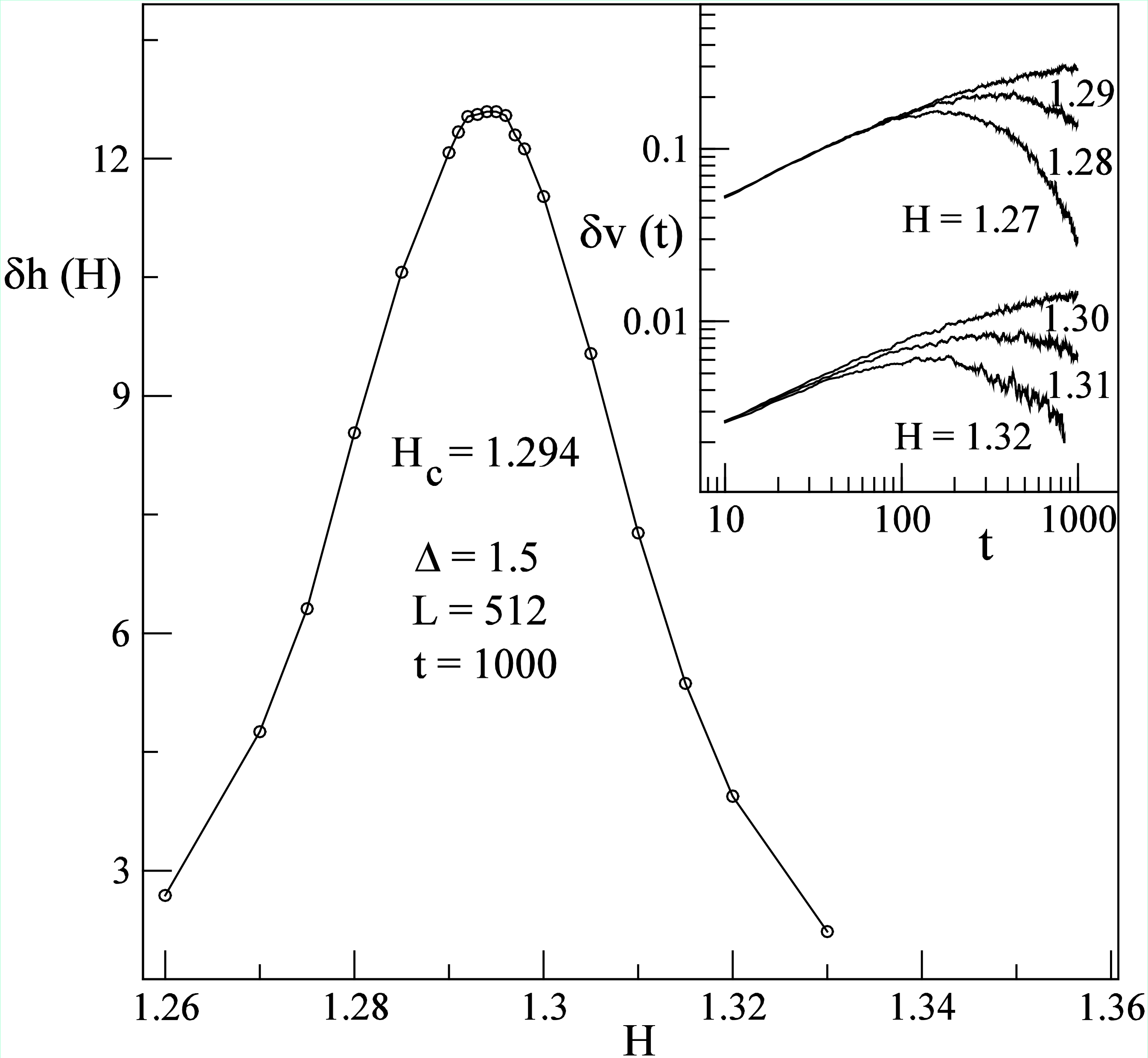
Overhang height function and velocity

$$\delta h(y,t) = h_e(y,t) - h_m(y,t),$$

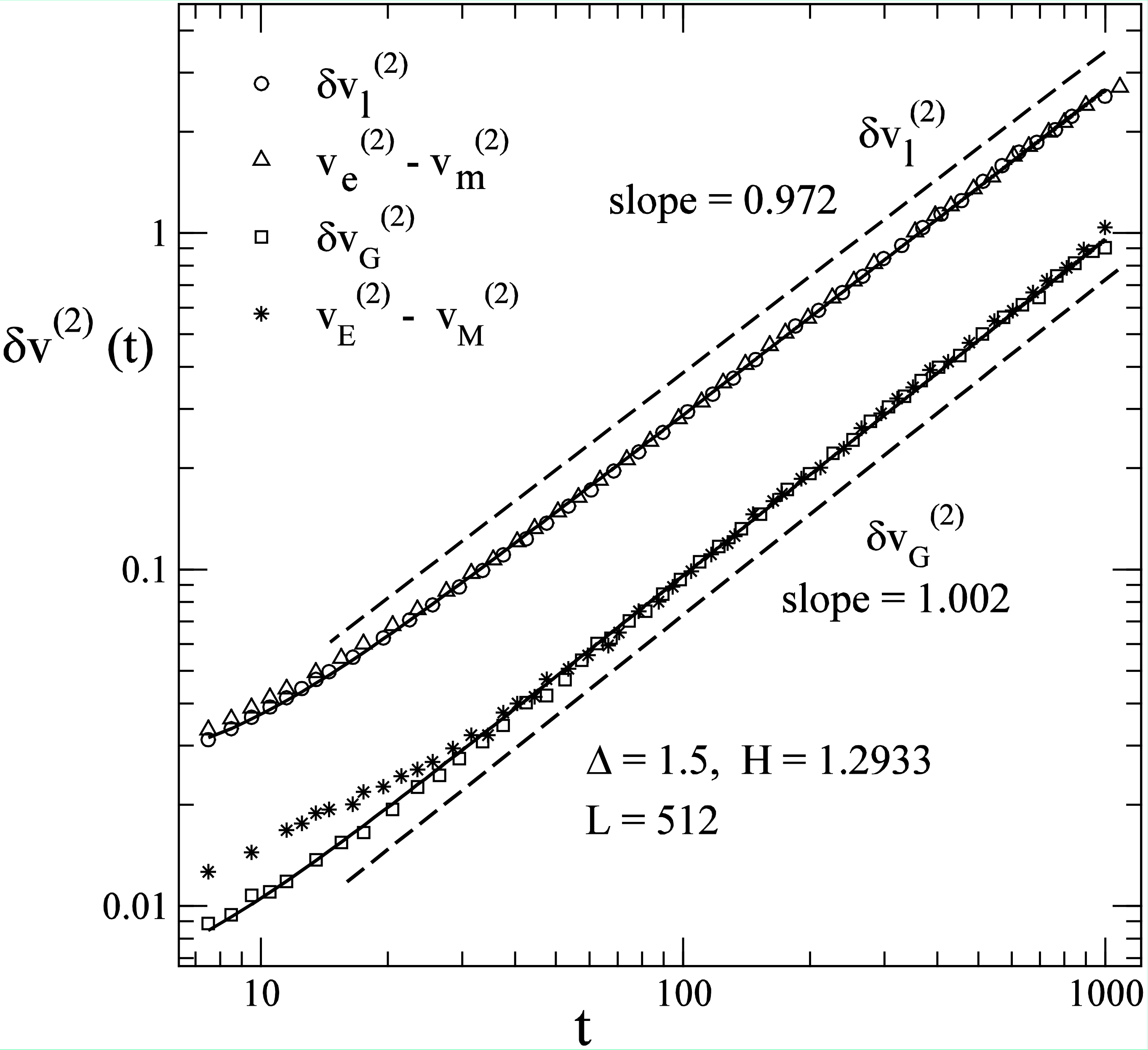
$$\delta v(y,t) = v_e(y,t) - v_m(y,t).$$

Overhang size and velocity reach maximum at the depinning transition point, and the velocity has no correlation in y direction

The overhang dynamics is also critical, and one needs another set of exponents to describe the overhang dynamics



δh (H)
H
$H_c = 1.294$
Δ = 1.5
L = 512
t = 1000
δv (t)
t
H = 1.27
1.28
1.29
H = 1.32
1.31
1.30



$\delta v_1^{(2)}$
$v_e^{(2)} - v_m^{(2)}$
$\delta v_G^{(2)}$
$v_E^{(2)} - v_M^{(2)}$
$\delta v^{(2)}(t)$
$\delta v_1^{(2)}$
slope = 0.972
$\delta v_G^{(2)}$
slope = 1.002
$\Delta = 1.5$, $H = 1.2933$
$L = 512$
t
1
0.1
0.01
10
100
1000

		QEW	Magnetization	Envelop	Overhang
$v(t)$	H_c		1.2933(2)	1.2913(4)	1.294(1)
	β	0.33(2); 0.33	0.295(3)	0.278(4)	
	ν	1.29(5); 1.33; 1.33(1)	1.02(2)	1.02(4)	$\gg 1$
	z	1.5; 1.53	1.33(1)	1.28(1)	1.11(1)
	θ				0.50(2)
$\omega^2(t)$	ζ	1.26(1); 1.25; 1.24	1.14(1)	1.14(1)	1.16(2)
$C(r,t)$	ζ	1.23(1); 1.25	1.13(1)	1.14(1)	
	ζ_{loc}	0.98; 0.92	0.735(8)	0.569(6)	
$v_M^{(2)}(t)$	λ		2.04(5)		
$v_m^{(2)}(t)$			3.06(3)		
$\delta v_G^{(2)}(t)$	α				0.501(3)
$\delta v_l^{(2)}(t)$					0.488(4)

ϕ^4 theory [EPL 99 (2012) 56001]

$$\mathcal{H} = \sum_i [\frac{1}{2}\pi_i^2 + \frac{1}{2}\sum_\mu (\phi_{i+\mu} - \phi_i)^2 - \frac{1}{2}m^2\phi_i^2$$

$$+\frac{1}{4!}g\phi_i^4 - h_i\phi_i - H\phi_i],$$

Initial states:

a perfect domain wall with v=0

zero kinetic energy π=0

the total energy differs from that of the ordered state only by the domain wall

Dynamics:

Hamiltonian equation conserved energy.

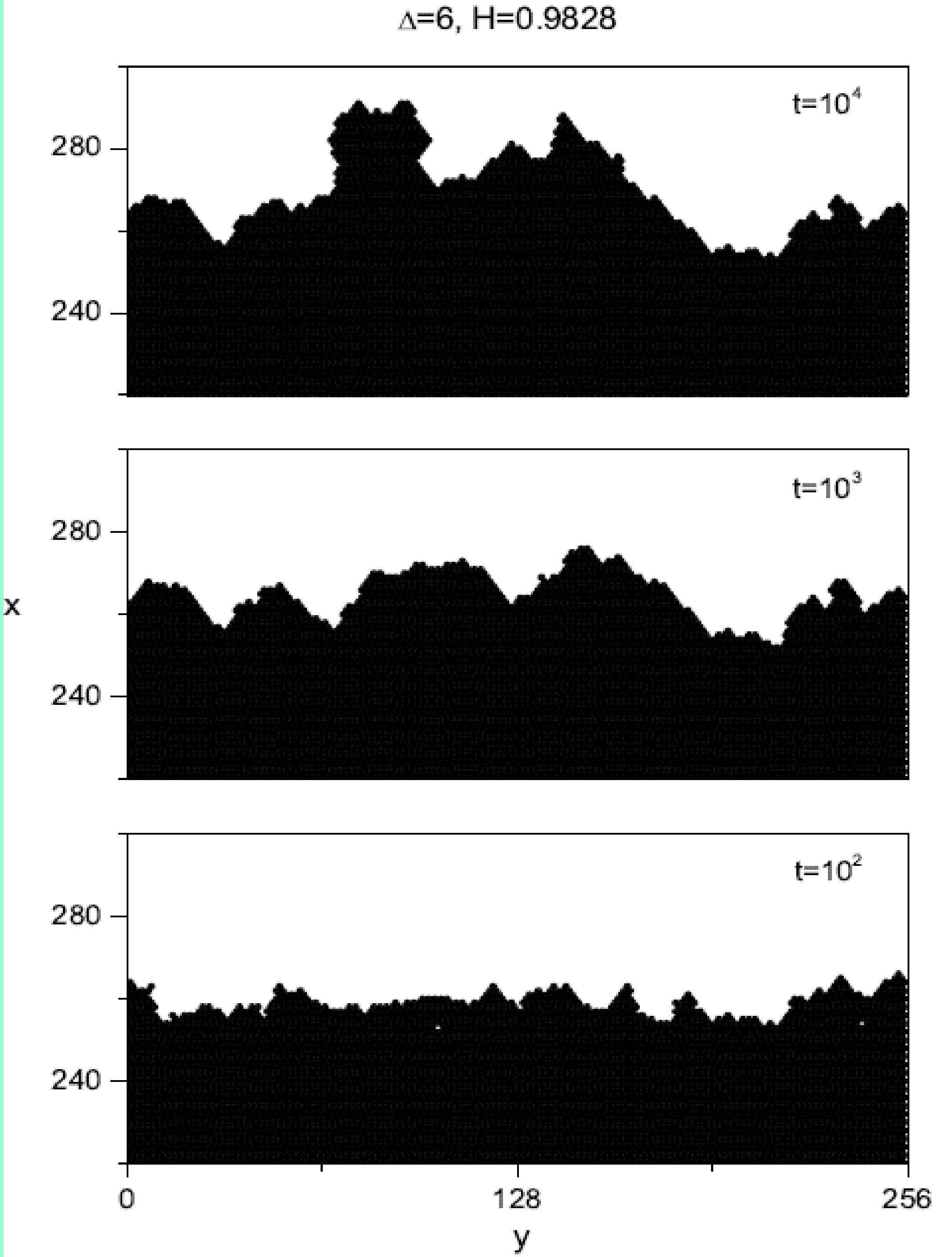
In the thermodynamic limit, it corresponds to the temperature close to zero, i.e., *T=0*.

$$\ddot{\phi}_i = \sum_\mu (\phi_{i+\mu} + \phi_{i-\mu} - 2\phi_i) + m^2\phi_i - \frac{g}{3!}\phi_i^3 + h_i + H$$

Time discretization:

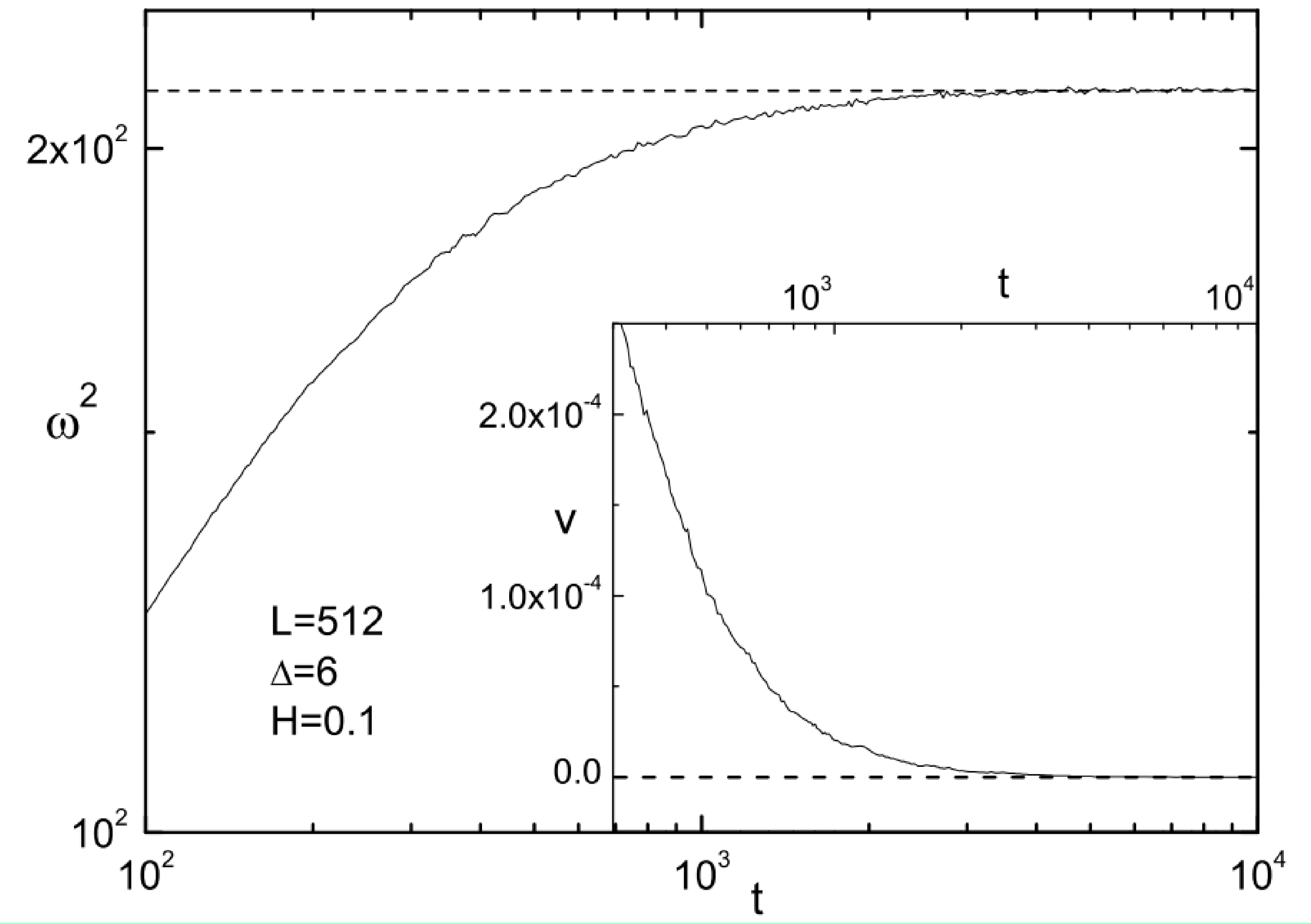
$$\ddot{\phi}(t) \rightarrow [\phi_i(t+\Delta t) + \phi_i(t-\Delta t) - 2\phi_i(t)]/(\Delta t)^2$$

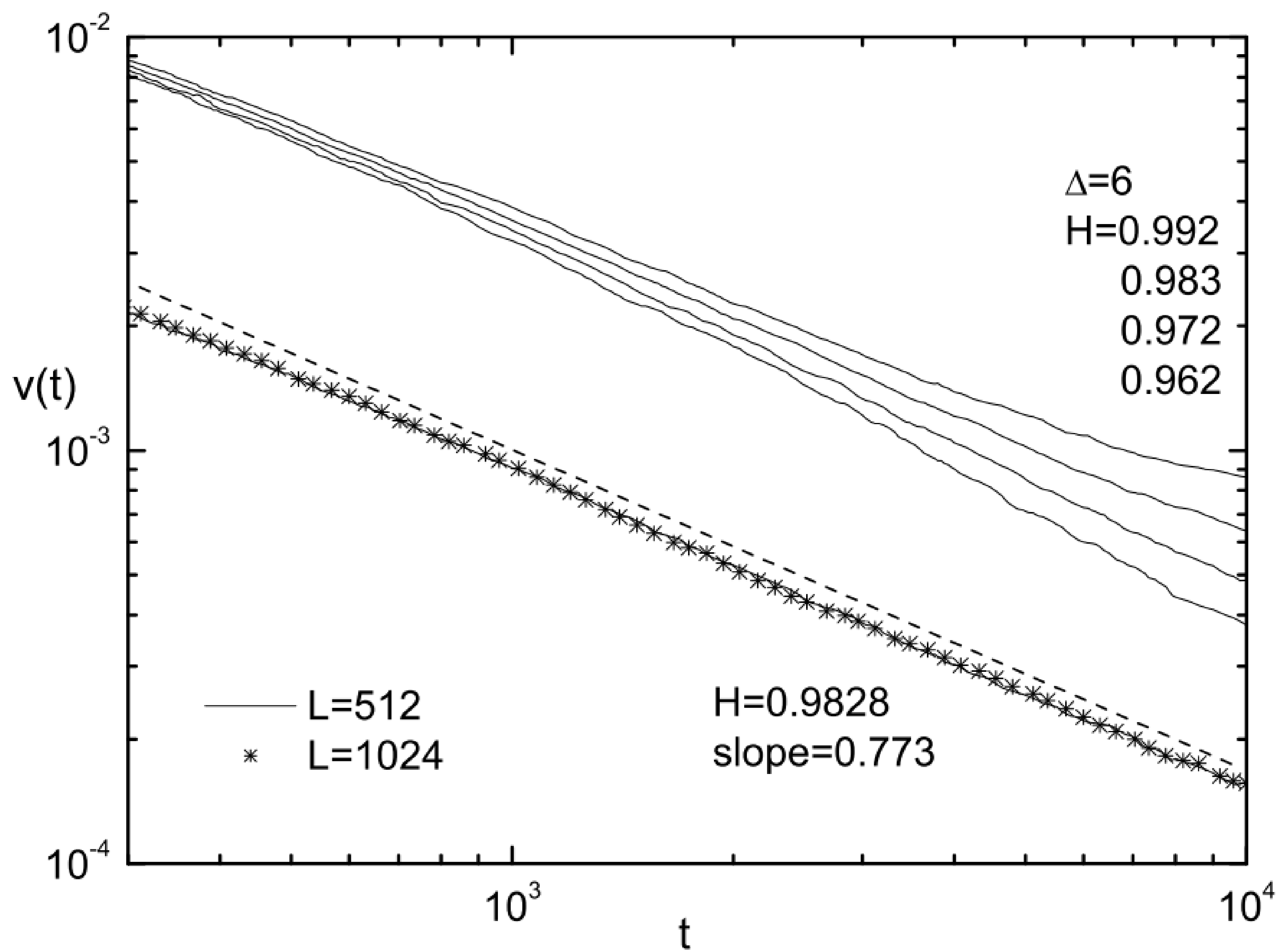
with $\Delta t = 0.02$



Δ=6, H=0.9828
t=10^4
t=10^3
t=10^2
280
240
x
0
128
256
y

Pinning phenomenon





10^{-2}
10^{-3}
10^{-4}
v(t)
10^{3}
10^{4}
t
Δ=6
H=0.992
0.983
0.972
0.962
L=512
L=1024
H=0.9828
slope=0.773

Main results

* ϕ^4 theory **does** describe pinning phenomenon and depinning transition

* The universality class is rather different from that of the Ising one, although ς_{loc} is similar, compatible with experiments

	β	ν	z	ζ	ζ_{loc}
ϕ^4	1.06(1)	0.70(1)	1.95(2)	0.865(5)	0.674(6)
DRFIM	0.295(3)	1.02(2)	1.33(1)	1.14(1)	0.735(8)
QEW	0.33(2)	1.33(4)	1.50(3)	1.25(1)	0.98(6)

*** Why are QEW and DRFIM in different universality classes?**

Answer: overhangs and islands

*** Why are DRFIM and ϕ^4theory in different universality classes? (In contrast to equilibrium phase transitions)**

Answer: at zero temperature, dynamic mechanisms for domain wall motion are restricted

Question: Langevin equation? Others?

The vector model

The XY model and Heisenberg model do not describe the pinning phenomenon, although a discrete vector model, i.e., the clock model does

How to simulate the domain-wall motion of vector magnets with Monte Carlo methods, especially the depinning transition, remains open

Landau-Lifshitz-Gilbert equation may be a solution

$$\dot{\vec{M}} = \gamma \vec{H}_{eff} \times \vec{M} + \alpha \vec{M} \times \dot{\vec{M}}$$
$$- g(\vec{J} \cdot \vec{\nabla})\vec{M} + \beta \vec{M} \times [(\vec{J} \cdot \vec{\nabla})\vec{M}]$$

p-state model with random-fields

Phys. Rev. E 86 (2012) 031129

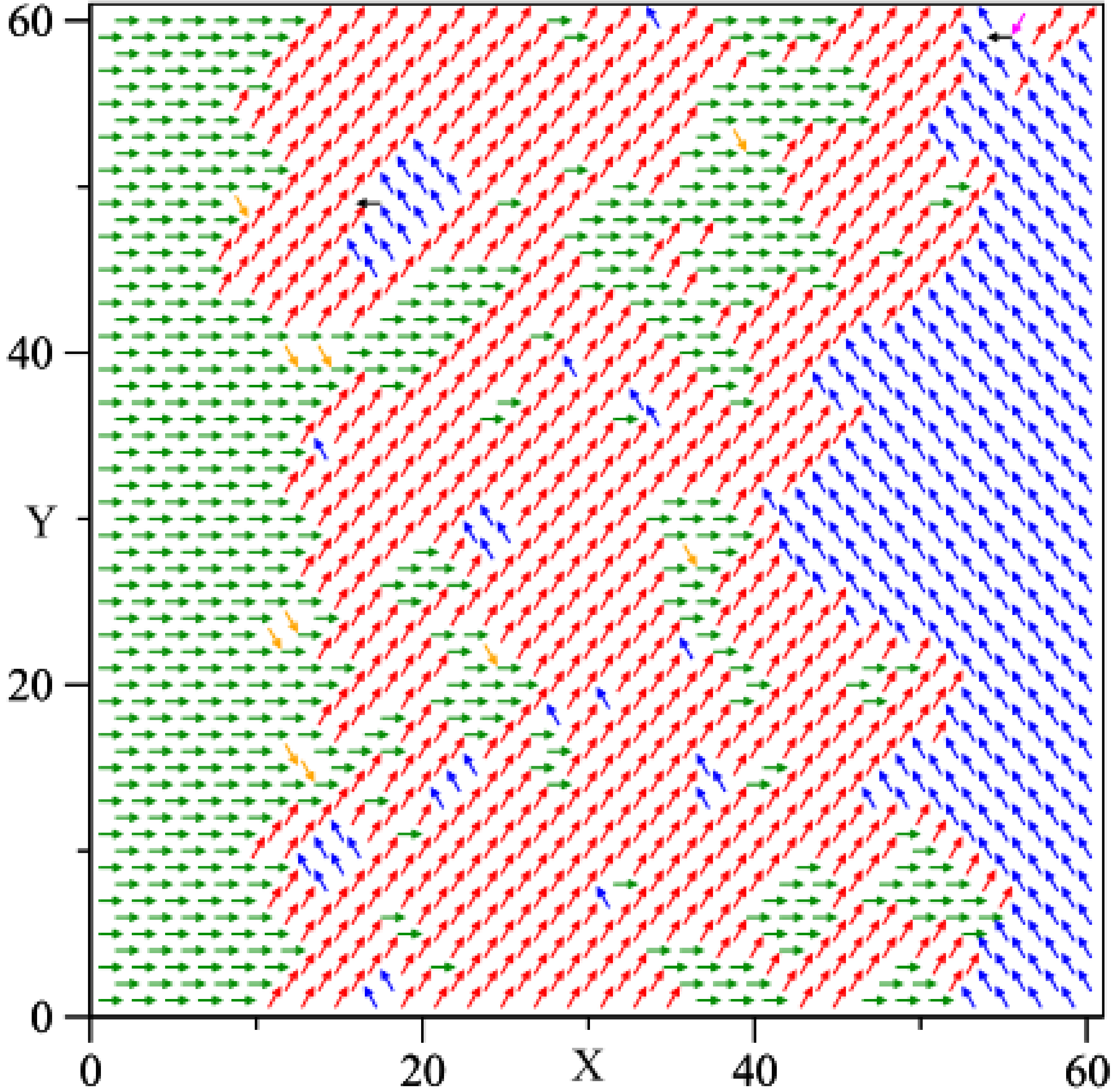
$$
\begin{aligned}
\mathcal{H} &= -J \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j - \sum_i (\vec{H} + \vec{h}_i) \cdot \vec{S}_i \\
&= -J \sum_{<ij>} \cos(\theta_i - \theta_j) - H \sum_i \cos\theta_i - \sum_i h_i \cos(\varphi_i - \theta_i)
\end{aligned}
$$

$\theta_i = 2\pi q_i/p$ with $q_i = 0, \ldots, p-1.$

Initial state:

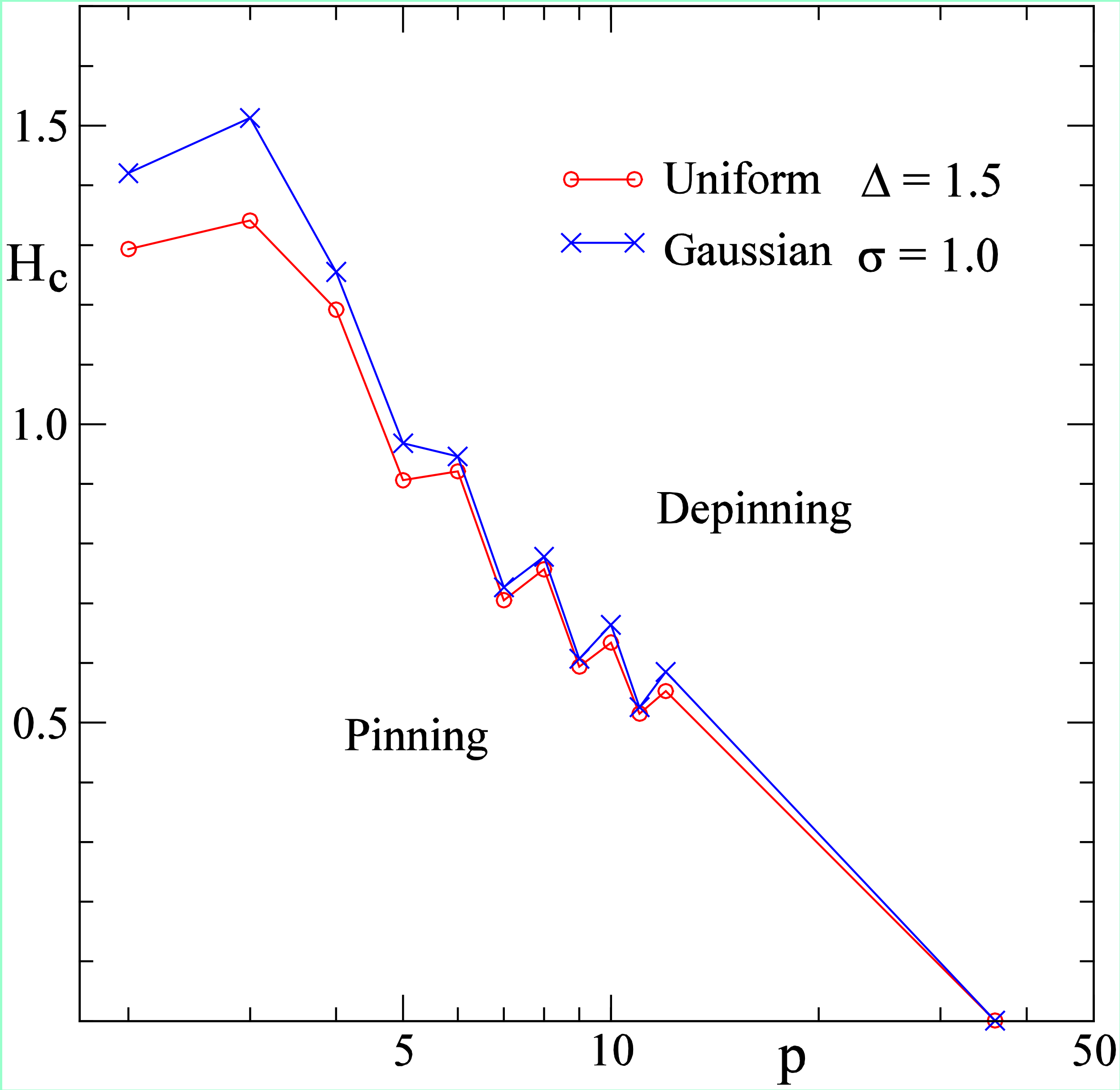
a perfect domain wall with *v=0*

now also with an orientation



(b) $\sigma = 1.5, H = 0.8020$

The *p* state clock model [Phys. Rev. E 86 (2012) 031129]



Summary

- Based on the short-time dynamic approach, we determine the critical exponents of the depinning phase transition for various lattice models with Monte Carlo and molecular dynamics simulations

- These lattice models and QEW equation are not in a same universality class, since overhangs and islands are important

- The numerical values of the local roughness exponent of the lattice models are compatible with the experimental results

Outline

I Introduction

II Depinning phase transition

III Relaxation-to-creep transition

Creep motion is activated by temperature, different from sliding driven by external fields

Characteristic behavior is

$$v \sim exp[-(H_c/H)^{\mu}/T]$$

Since *v* is small, measurement in the steady state is difficult

Experiment: μ **is between 1/4 and 1**

QEW: $\mu = 1$

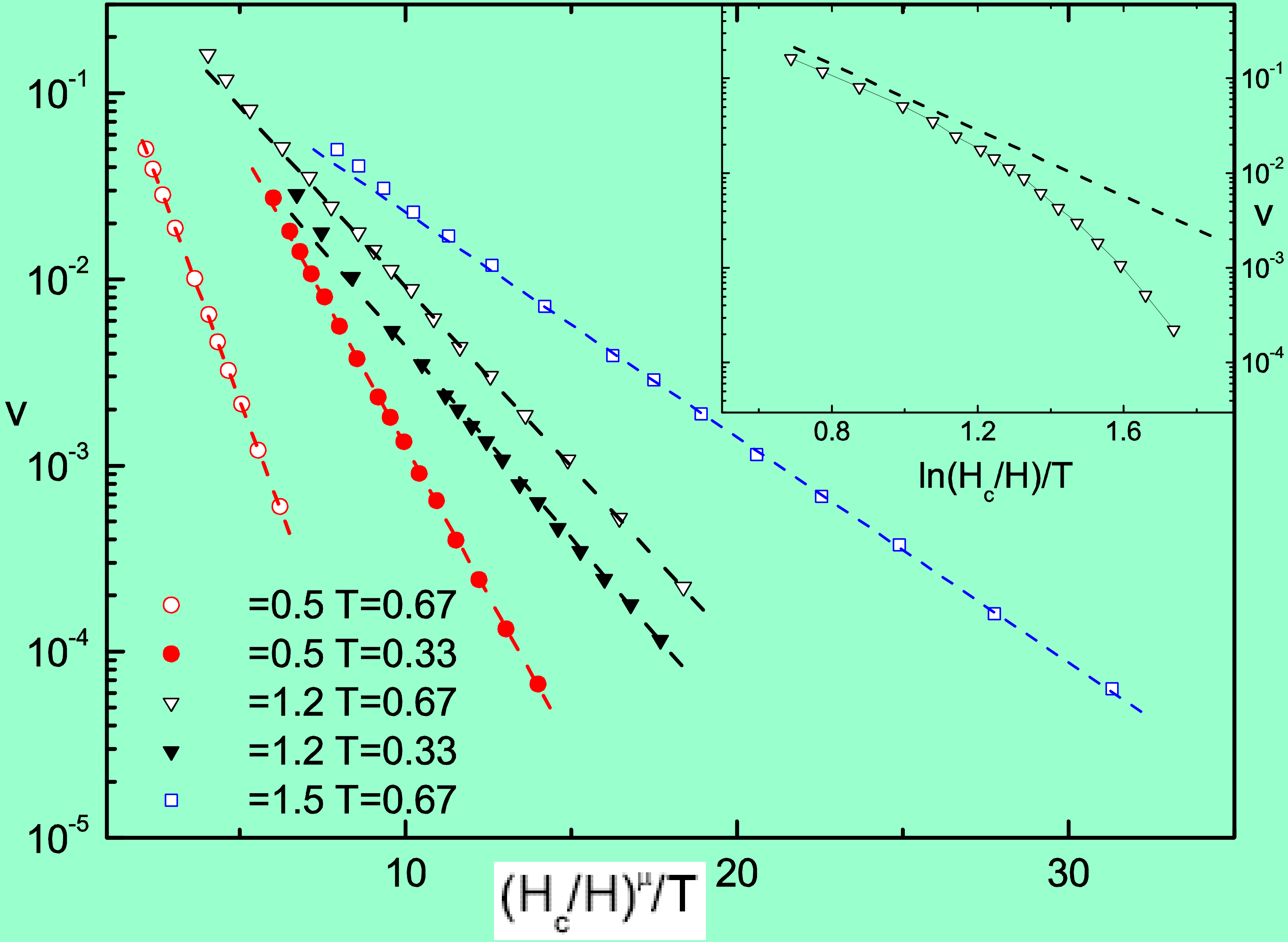
Random-field Ising model

EPL 98 (2012) 36002

$$\mathcal{H} = -J\sum_{\langle ij\rangle} S_i S_j - \sum_i (h_i + H) S_i .$$

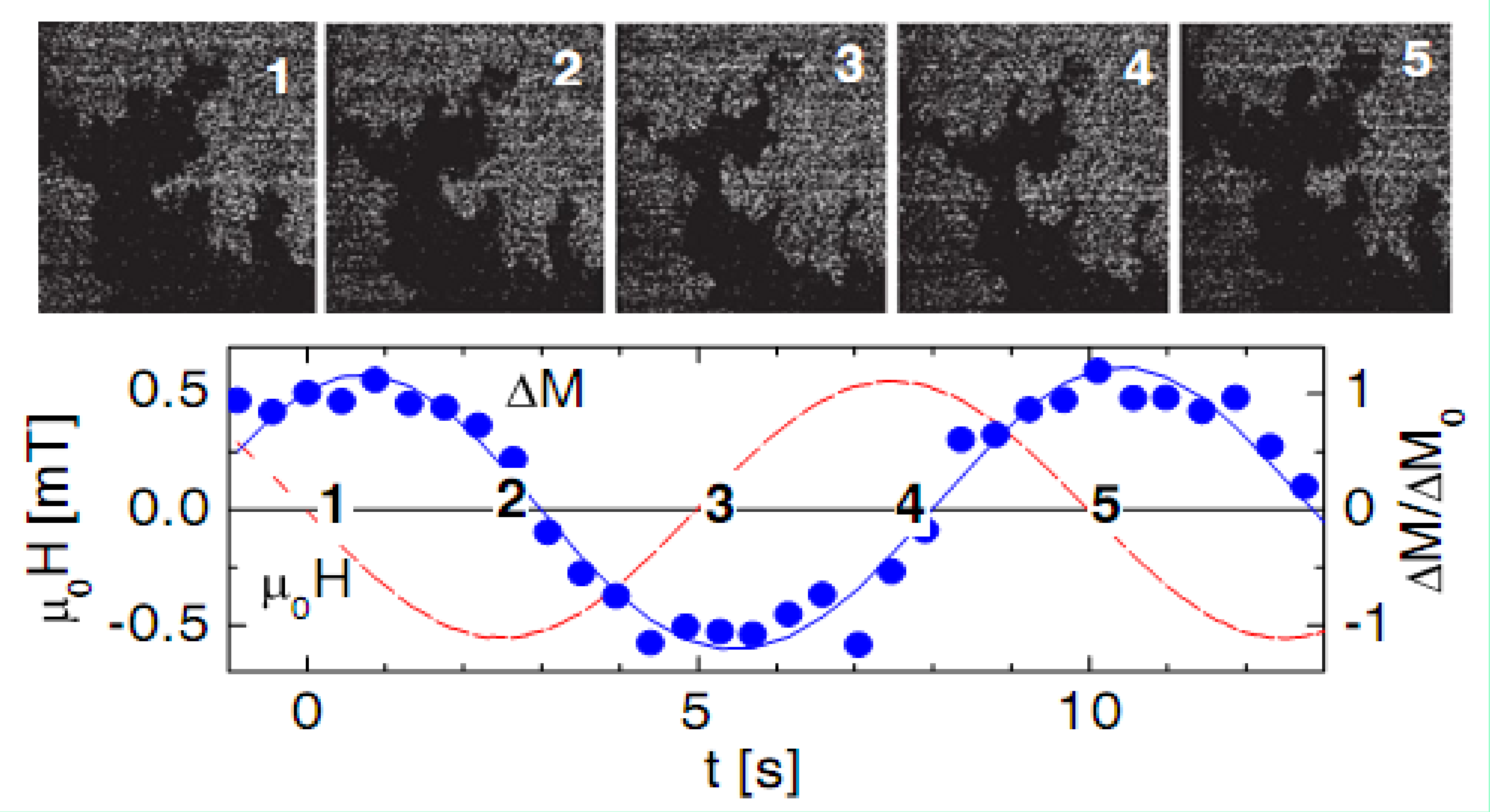
Monte Carlo simulation at *steady state*

	$\Delta = 0.5$		$\Delta = 1.0$		$\Delta = 1.5$
T	0.67	0.33	0.67	0.33	0.67
μ	0.63(5)	0.59(4)	0.98(4)	0.90(5)	1.14(5)
ψ	0.69(5)	0.65(6)	1.01(2)	0.95(4)	1.28(3)
ζ	0.85(2)		0.98(3)		1.15(3)
θ	0.71(4)		0.96(6)		1.31(6)
$\psi/(2-\zeta)$	0.60(5)	0.57(6)	0.99(5)	0.93(7)	1.51(9)



V
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10
20
30
$(H_c/H)^{\mu}/T$
=0.5 T=0.67
=0.5 T=0.33
=1.2 T=0.67
=1.2 T=0.33
=1.5 T=0.67
0.8
1.2
1.6
$\ln(H_c/H)/T$

Relaxation-to-creep phase transition



Experiments of ultrathin magnetic films

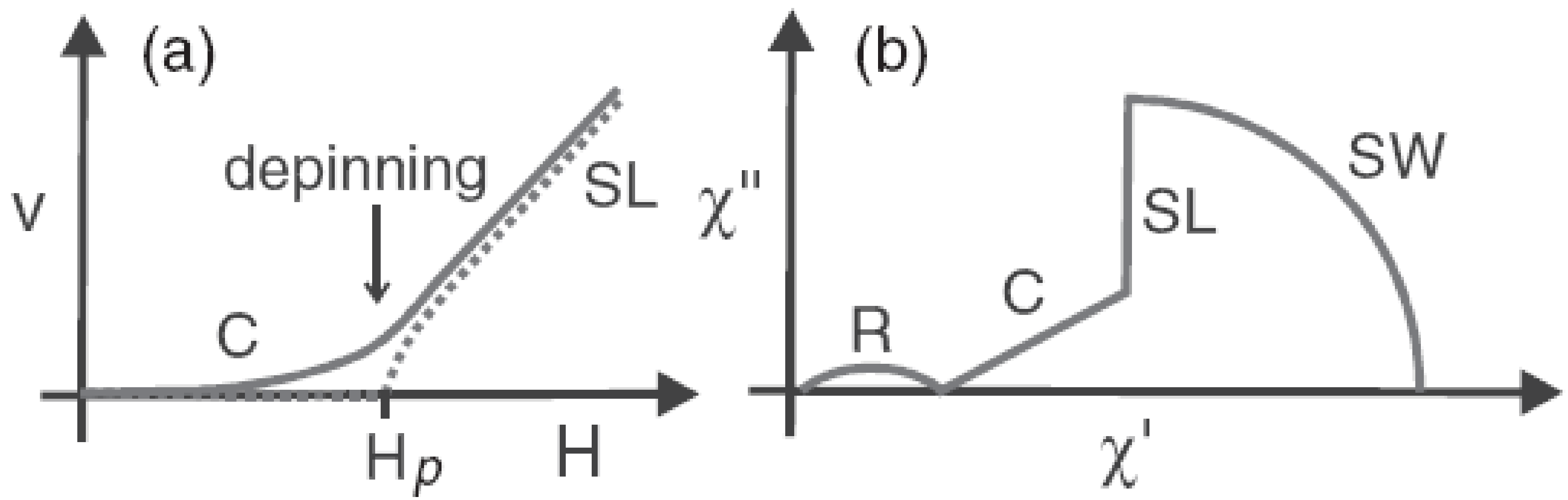
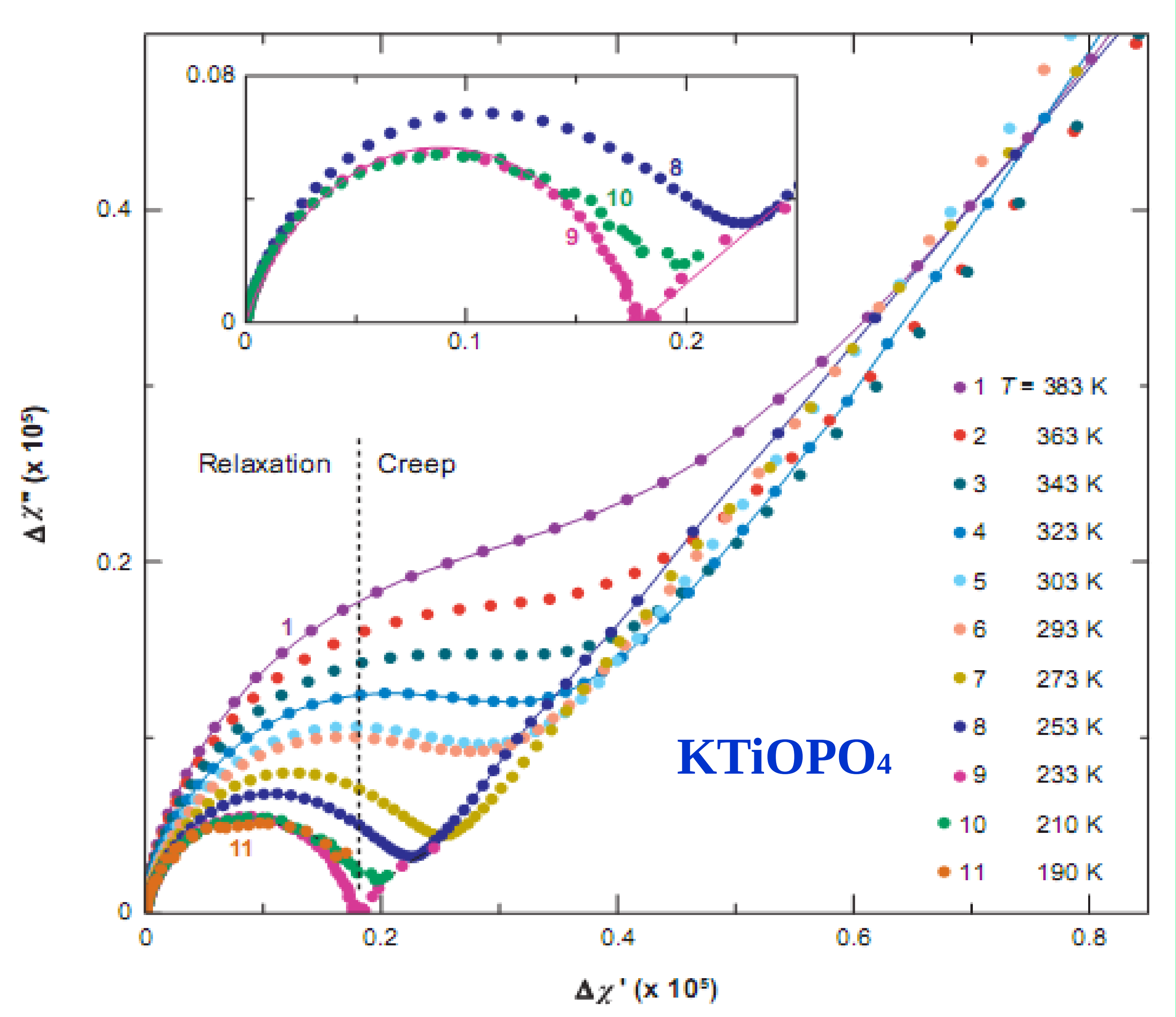


FIG. 1. (a) Schematic plot of the DW velocity, v vs dc field H, which does not report on the high-field behavior, exhibiting depinning at $H = H_p$ and slide (marked as SL) at $T = 0$ (broken line) and—additionally—creep (C) at $T > 0$ (solid line). (a) Schematic Cole-Cole plot of the susceptibility components, χ'' vs χ', due to a randomly pinned DW in ac driving fields, exhibiting relaxation (R), creep (C), slide (SL), and switching (SW).

Cole-Cole plot for Relaxation-to-creep transition



Random field Ising model with ac external field

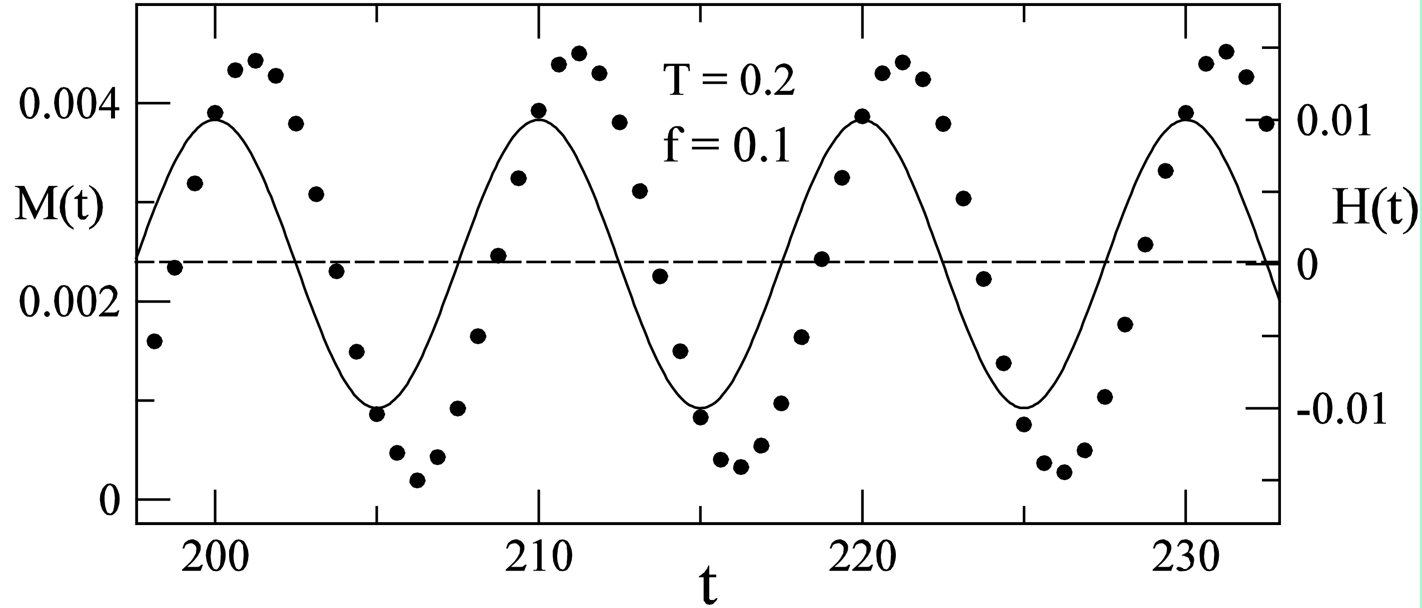
$$\mathcal{H} = -J\sum_{\langle ij\rangle} S_i S_j - \sum_i (h_i + H) S_i .$$

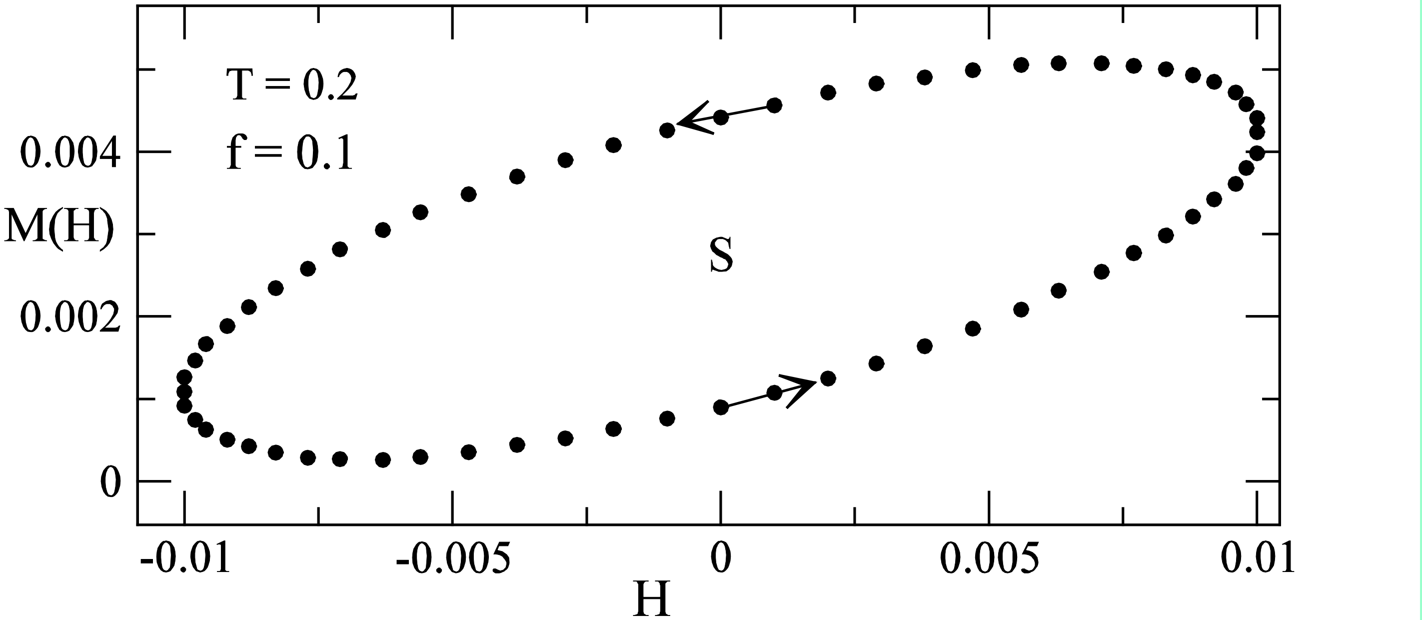
with an ac external field $H = H_0 \cos(i\omega t)$
at low temperatures
at low field H_0 = 0.01

The stationary state

N.J. Zhou, **B. Zheng** and D.P. Landau,
EPL 92 (2010) 36001

The stationary state





The complex ac susceptibility

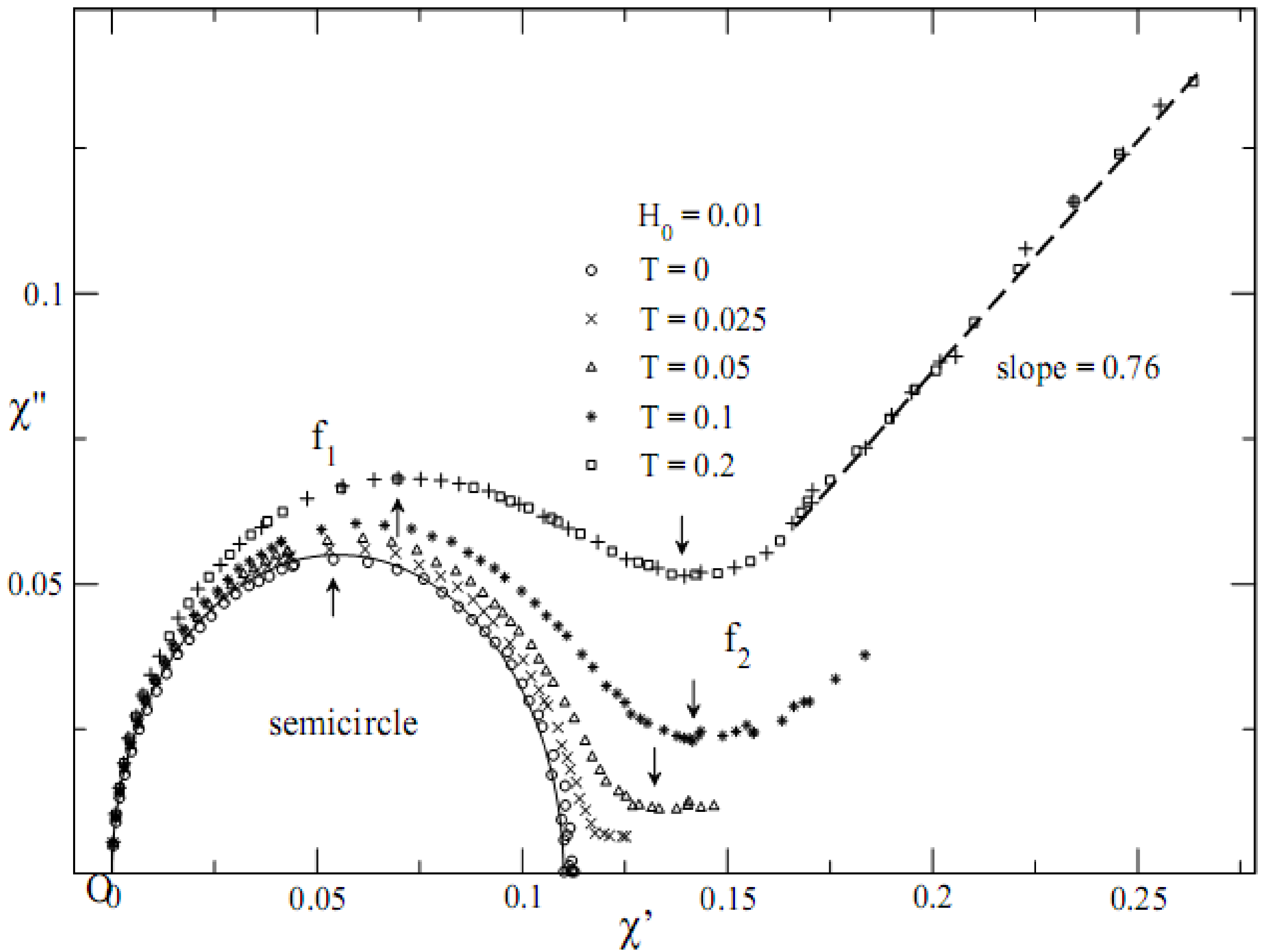
$$\chi'(\omega) = \frac{1}{H_0 T} \int_0^T dt M(t) \cos(i\omega t)$$

$$\chi''(\omega) = \frac{1}{H_0 T} \int_0^T dt M(t) \sin(i\omega t)$$

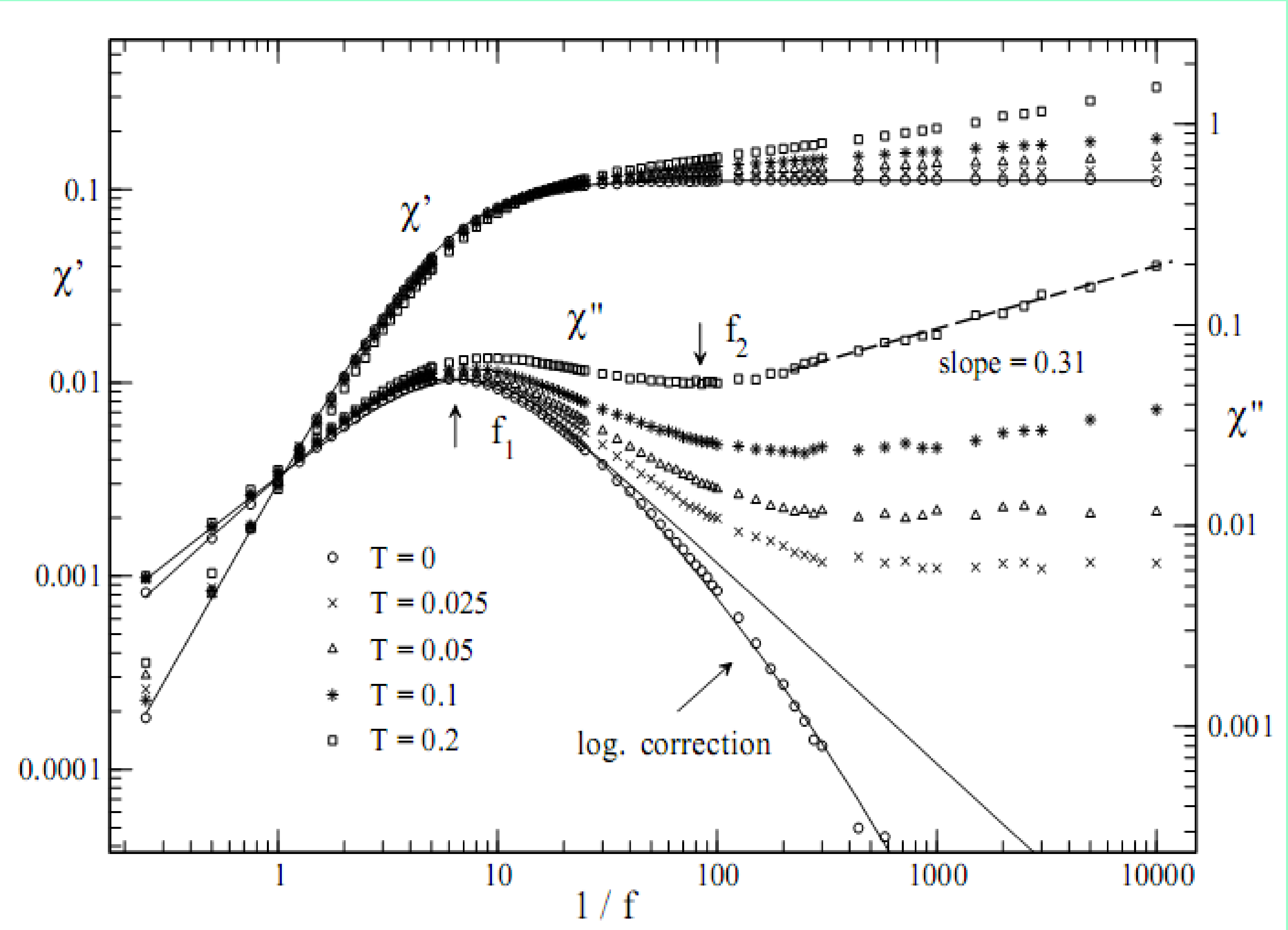
The behavior of susceptibility χ

In relaxation regime: $\chi(\omega) = \chi' - i\chi'' = \dfrac{\chi_\infty}{1 + i\omega\tau}$

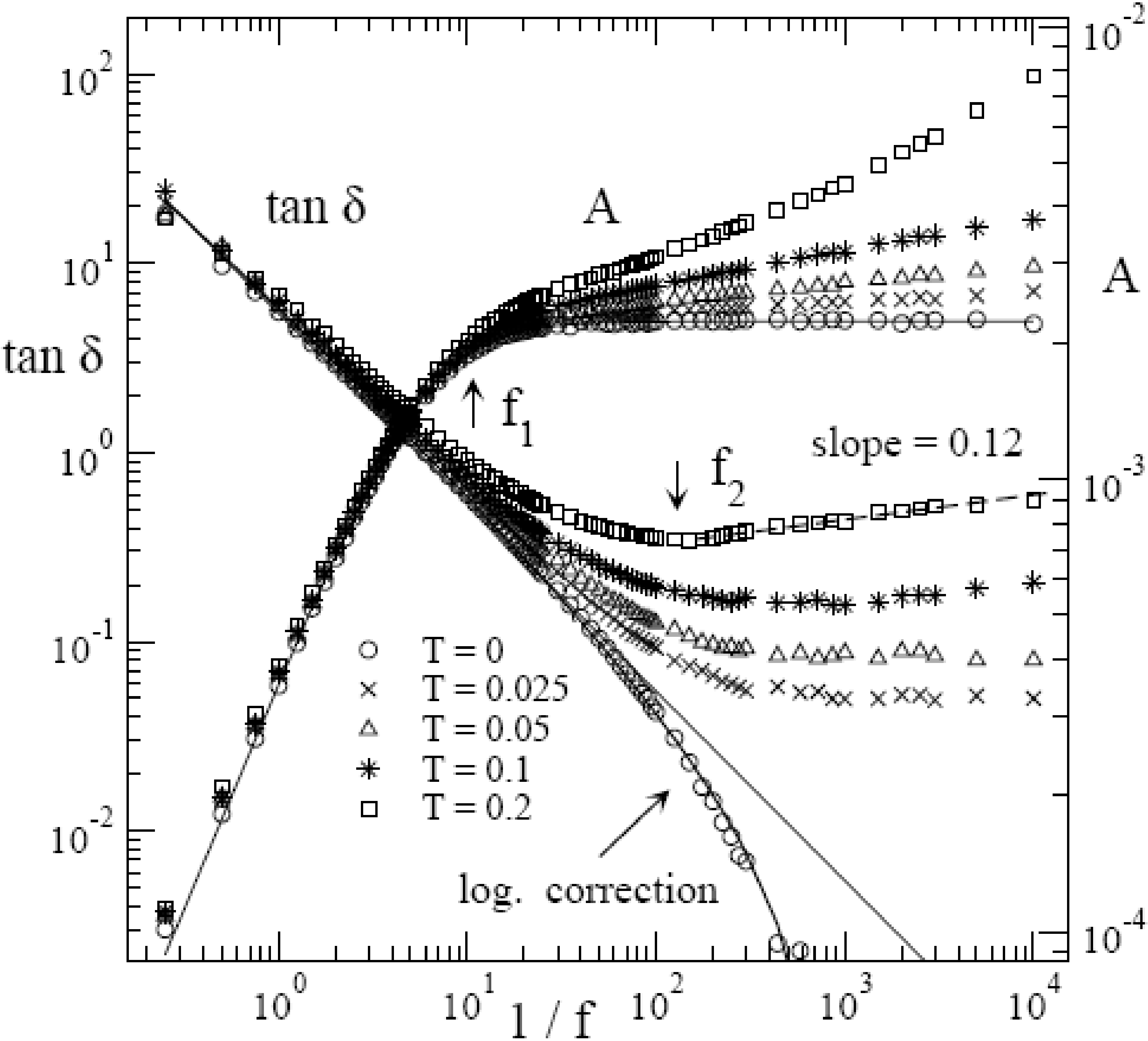
In creep regime: $\chi(\omega) = \chi_\infty \left(1 + (i\omega\tau)^\beta\right)$



$H_0 = 0.01$
T = 0
T = 0.025
T = 0.05
T = 0.1
T = 0.2
slope = 0.76
f_1
f_2
semicircle
χ"
χ'
O
0
0.05
0.1
0.15
0.2
0.25



χ'
χ''
χ'
χ''
f1
f2
slope = 0.31
T = 0
T = 0.025
T = 0.05
T = 0.1
T = 0.2
log. correction
0.1
0.01
0.001
0.0001
1
0.1
0.01
0.001
1
10
100
1000
10000
1 / f


tan δ
A
tan δ
A
f1
f2
slope = 0.12
T = 0
T = 0.025
T = 0.05
T = 0.1
T = 0.2
log. correction
1 / f

Summary

- **With Monte Carlo methods, the creep motion and relaxation-to-creep transition of a domain wall is investigated for the random field Ising model**
- **For the creep motion, the exponent μ changes with the disorder strength, compatible with the experimental results**
- **For the relaxation-to-creep transition, the Cole-Cole plot in experiments is obtained**

Outlook

How to simulate the depinning transition and relaxation-to-creep transition in the domain-wall motion of vector magnets with the Landau-Lifshitz-Gilbert equation