Cascading dynamics in complex quantum networks

Liang Huang[1,2] and Ying-Cheng Lai[2,3,4]
[1]*Institute of Computational Physics and Complex Systems and Key Laboratory for Magnetism and Magnetic Materials of MOE, Lanzhou University, Lanzhou, Gansu 730000, China*
[2]*School of Electrical, Computer, and Energy Engineering, Arizona State University, Tempe, Arizona 85287, USA*
[3]*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*
[4]*Institute for Complex Systems and Mathematical Biology, School of Natural and Computing Sciences, King's College, University of Aberdeen, Aberdeen, United Kingdom*

(Received 14 January 2011; accepted 18 May 2011; published online 28 June 2011)

Recently, small-scale quantum communication has been realized by transporting entangled photons, rendering potentially feasible quantum networks on large scales. We propose a class of quantum networks comprising quantum repeaters for long-distance information transport and local networks of fibers and switches. As the transmitting capability is limited by the node bandwidth, photon loss can occur through the dynamical process of cascading when the network is under external perturbations. We obtain results that can be used to guide quantum network design to minimize the photon loss. © *2011 American Institute of Physics*. [doi:10.1063/1.3598453]

Quantum communication is capable of providing an unprecedented level of security against eavesdropping.[1–4] Recently, a two-node quantum communication system has been realized where a single photon is transmitted and retrieved through an optical fiber.[5–8] During the processes, the quantum characteristics of the photon carrying the information are well preserved. It is conceivable that multi-party quantum communication is needed in the future, rendering a relevant and important systematic exploration of *quantum networks* in terms of topology, dynamics, and security. To make long-distance communication feasible, a device capable of transferring quantum information is necessary. Recent technological advances have made such a device possible, the *quantum repeaters*. Specifically, a quantum repeater contains a light-matter interface, a quantum memory (e.g., in the form of an ensemble of atoms), and elementary quantum gates. Upon receiving a photon, a quantum repeater stores it in the memory and produces a new photon with the same quantum characters. The repeater can also introduce desirable delays. A network of quantum repeaters can then transport an entangled photon carrying, e.g., a quantum key, to any part of the network.[9–23] While at present, no nation-wide commercial quantum network exists, one can envision the emergence of large-scale quantum communication system in the near future. Timing is thus proper to start investigating the feasibility of quantum networks. Here, we propose a class of hybrid quantum networks for large-scale quantum entanglement distribution based on the physics of quantum communication. There are two components in such a network: quantum repeaters for long-distance information transport and local networks of fibers and switches. As in many other existing infrastructure networks, when a quantum network grows, it tends to become more complex. As a result, complex-network topologies can be a natural consequence of the network construction. To exemplify the fundamental issues that need to be addressed before constructing realistic complex quantum networks, we focus on one phenomenon: photon loss as triggered by external perturbations and internal traffic fluctuations. We propose a soft cascading process to model the photon loss. We find, surprisingly, that the degree of photon loss can reach maximum when the network is neither sparsely nor densely connected. The phenomenon can in turn be used to guide network design to mitigate photon loss and enhance the communication efficiency of quantum networks.

We first provide a brief background description of quantum communication. Entanglement distribution among distant nodes is the key to quantum communication. Because quantum decoherence and attenuation can occur for links whose physical distances are large, so far the maximum distance for optical-fiber based quantum communication is about 50 km.[8] Thus to make long-distance entanglement distribution feasible and large scale quantum communication possible,[9,10] a device capable of purifying and relaying quantum entanglement is necessary. The basis is a quantum interface between light and matter, which serves as the quantum channel and the quantum node, respectively. Protocols have been proposed using both the cavity quantum electrodynamics approach[11–13] and atomic ensembles.[10,14] Experimental realizations of such protocols have been developed lately.[5,15–20] This builds the basic blocks for *quantum repeater*,[9,10,21,22] which has been demonstrated experimentally recently using atomic ensembles.[23] Upon receiving a photon, a quantum repeater stores it in the memory (atomic ensembles) and produces a new photon with the same quantum characters on demand. The repeater can also introduce desirable delays. A network of quantum repeaters can then establish quantum entanglement between any parts of the network, which can be used to set up quantum key sharing.

Based on quantum repeaters, a trusted relay network can be constructed.[8] In an idealized situation, one can imagine using the repeaters as routers and connecting them to form a quantum entanglement network, where each node (repeater) can transport entangled quantum states to any other node by following a path consisting of repeaters and fibers. The current internet protocols for selecting paths can be adopted. This network of quantum repeaters thus forms the backbone of the quantum key distribution (QKD) network, where endpoint users can be attached to any node and share quantum keys with users attached to other nodes in the network. The number of links required is of the order of $O(N)$. The network can be constructed across vast physical distances. Redundancies can be built into the network so that disabling a fraction of links will not affect the communication capability of the network as keys can be redirected through the alternative paths. The main issue, however, concerns security. Since the quantum states of entangled photons are stored in the memories of the repeaters, security will be compromised when repeaters are under attack.

To ensure security, one can imagine an untrusted network consisting of photonic switches,[8] without any quantum repeaters. A single switch simply connects an incoming fiber with an outgoing fiber, and it does not assess or influence the quantum state of the information-bearing photon. For a pair of nodes (endpoints), the untrusted network provides a path of fibers and switches to transmit photons. Insofar, as the destination endpoint receives the photon, the key is confidential. A photonic switch can be built from micro-electro-mechanical (MEM) mirror arrays. Some proper protocol for the switches sets up various end-to-end paths across the network to avoid the paths that are susceptible to attacks or eavesdroppings. While the maximum distance along any path should be less than the optical attenuation distance of fibers, the network is secure and is especially suited for local-area QKD applications. There is also advantage in terms of cost as MEM devices can be fabricated in large quantities.

We now propose a class of large-scale quantum networks, taking into consideration the unique characteristics of quantum communication and network security. Our proposed network is of a "hybrid" type, comprising the quantum-relay and the switch-and-fiber components embedded within each other over the physical space that supports the quantum network. In our construction, the untrusted network component is responsible for urban-scale QKD as most cities have their diameters smaller than 50 km. To serve a larger area, a relay quantum network can be incorporated, which serves as the backbone and bridges distant users from various untrusted local networks. Our proposed scheme is demonstrated in Fig. 1(a). In general, the switches should be in an "off" state so that the fibers are not connected. When two end users need to share quantum keys, a suitable protocol can be used to trigger the establishment of a path between the users. Physically, this corresponds to turning on a set of switches. Depending on the distance between the users, the path may or may not contain any repeaters. If a link on the path fails or is disabled by eavesdropping activities, a different path can be set up to transport the entangled photons.

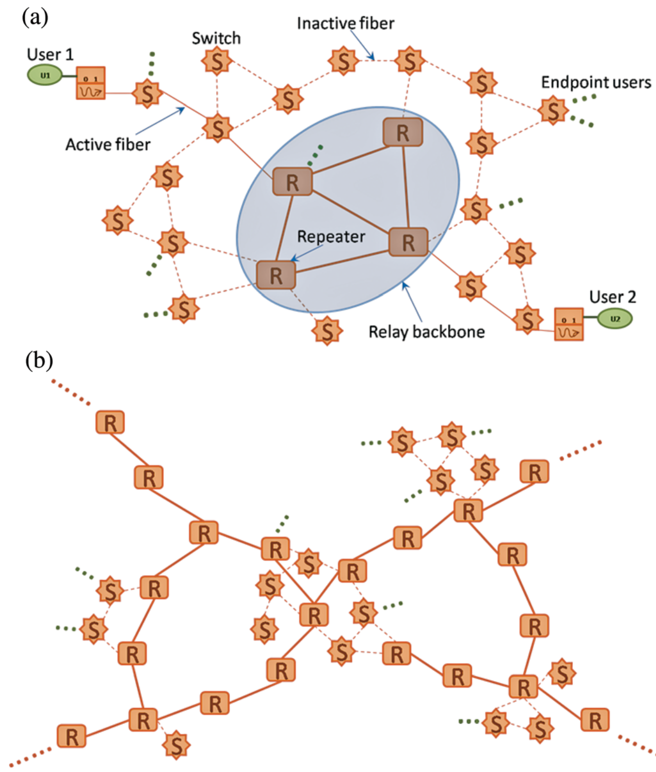

FIG. 1. (Color online) (a) Proposed hybrid QKD network on large scales. Each switch or repeater can have many endpoint users and those attached to repeaters have high priority in terms of security. (b) Illustration of a small portion of a large-scale hybrid network. A component of the relay network can connect directly with another component or it can use controlled switches to activate a connected path when needed. Each repeater can have many switches attached, which can connect to more switches or directly to endpoint users. Well-protected, high-priority endpoint users can be connected directly to a repeater.

Figure 1(b) shows our proposed scheme of network for inter-city QKD on a global scale, where the repeaters and fibers connecting them form the basic relay backbone for entangled photons. Note that transporting entangled photons over vast distance, e.g., across different continents, can potentially be accomplished via satellite communication.[8] Certain part of the relay network is protected to ensure security. In a local (urban) area, switches directly linked to repeaters can connect more switches and/or endpoint users. To reduce cost, various junctions in the relay network can be realized by small switch networks that are connected only when necessary, depending on the protocol. High-priority endpoint users requiring a high level of security can be directly connected to various repeaters.

There are many fundamental issues associated with the proposed quantum networks such as the structure, dynamics, information flow, traffic protocol, robustness, and security, etc., which need to be addressed to make large-scale quantum networks a reality. Note that a dynamic protocol for the switches and repeaters to operate coherently to control the flow of photons and direct each photon to a proper descendent is still far from a clear understanding. Here, we assume such a protocol of transmitting entangled photons is available and for the remaining of this paper, we shall focus on one issue: photon loss. This issue, when neglecting the technical details of how the entangled photons are received and transmitted, shares much of the underlying physics with an ordinary system of data or information transmission.

Note that our proposed hybrid quantum network transports entangled photons, but a node (repeater or switch) usually has a finite bandwidth B. If a node receives too many photons at a given time, it can be jammed. For a switch-type of node, some photons will be lost. While some proper routing protocol can divert some information-carrying photons to other nodes, more nodes will get jammed if they are operating near the limits of their respective bandwidths. The jamming can thus spread on the quantum network in a manner mimicking a cascading process. In contrast to a *hard* cascading process reported extensively in the complex-network literature where failed nodes are disabled and are completely cut off from the network,[24–27] here it is a *soft* type cascading process leading to only loss of photons, because jammed nodes are still expected to work within their respective bandwidths.

To construct a computational model for photon loss, we assume that the photons travel along their respective shortest paths,[28] i.e., paths having minimum sum of weights. Generally, both the links and the nodes can have weights due to different physical procedures. For example, links can have weights due to the difference in the lengths of the fibers or their bandwidths. Nodes can also have weights due to the abilities and operating times to transmit a photon from an incoming link to an outgoing link. To facilitate analysis, we assume that the weights for the links are identical and have the numeral value of unity, while the weights of the nodes are dynamical and can be adjusted to mitigate the photon flow. The initial weights for the nodes are assumed to be zero.

During each time unit τ, we assume that there are β photons to be communicated between each pair of nodes. Since the photons are transmitted along the various shortest paths, in the steady state, within a time unit, the number of photons arriving at node i is βb_i, where b_i is the number of shortest paths passing through this node. To ensure normal operation, the bandwidth B_i of the node should satisfy the condition $B_i \tau > \beta b_i$. Thus, to avoid any bottleneck effect for efficient transportation of entangled photons, the bandwidth B_i should be proportional to b_i. In general, we can set $B_i = C b_i$, where C is the bandwidth parameter. Here, we have neglected the detailed processes for the repeaters and switches to transmit a photon, but assumed that each node, disregarding its type, can be built with sufficient bandwidth as required by the topological connection. As the value of β approaches $C\tau$, jamming can occur due to random traffic fluctuations. When this happens, we increase the weight of this node by one to mitigate the load running through it. However, this can lead to load increase and consequently jamming at nodes that are originally not jammed. To simulate this process, we apply a perturbation to the node possessing the largest bandwidth and increase its weight by a unit to trigger a cascading-like process. In the steady state, the number N_a of nodes that have been jammed in the process, i.e., nodes with weight larger than one, characterizes the degree of photon loss. In particular, for small photon generation rate β, cascading is unlikely to occur so that photon loss is not severe, resulting small N_a values. As β is increased, cascading can occur on part of the network, leading to increased values of N_a. For large photon generation rate, i.e., for $\beta \lesssim C\tau$, N_a can be comparable to the network size N, as almost all nodes can be affected by a single perturbation in this case, resulting in a significant photon loss.

For the particular topic of photon loss, we shall focus on a basic issue: for a given type of network structure, how the data generating rate, and the number of links affect the scale of soft cascading of photon loss. Intuitively, one would anticipate that, as the link density of the quantum network is increased, the average network distance is decreased, and the average number of shortest paths through a node decreases. Thus, photon loss due to cascading can be less severe for denser linkage. However, we find that photon loss tends to be maximized for a certain value of the link density.

As a QKD network grows, it will typically evolve into a complex networked structure. To be specific, since the QKD network is typically constructed upon the 2D surface of cities, it will have the feature of certain geographical networks, e.g., a backbone of locally regular structure with a small number of long-range shortcuts, resulting in small-world networks.[29–31] Since the distribution of usage is typically heterogenous, the QKD network topology can become increasingly heterogeneous in that a small subset of nodes can have substantially more links than others, leading to the scale-free topology.[32] In addition, due to the geographical distribution of the users (e.g., in various cities), the resulting quantum network can have a clustered structure, where each city is a densely connected cluster of quantum fiber and switch network, and different clusters are connected by relay lines between the cities. Thus in the following, we shall examine the features of small world, scale free, and clustered structure for the effects of photon loss, respectively.

To illustrate our finding, we first employ the small-world topology. The regular backbone network is assumed to have a ring-like structure of N nodes, where each node is connected to m nearest-neighbor nodes, and the number of random shortcuts is N_e. In our simulation, we assume, for convenience, that both the bandwidth parameter C and the time unit τ are unity. When the network reaches a steady state in the sense that load redistribution terminates, we count the total number of N_a of affected nodes with their weights changed.

Representative results are shown in Fig. 2. From Fig. 2(a), we see that when the photon generating rate is small, the perturbation can be effectively absorbed by the network so that the number of affected nodes is small. This is so because after a node is perturbed, some shortest paths passing through it will be detoured to its neighboring nodes. Since β is small, the load increase at these nodes is also small as compared with their capacities. As a result, the disturbance can be absorbed completely by these neighboring nodes without triggering a large scale soft-cascading process. When β is relatively large, the perturbation can result a cascading that affects many nodes in the network. In particular, for $\beta \lesssim 1 (= C\tau)$, the system is near its critical point in the sense that most of the nodes operate near their capacities. In

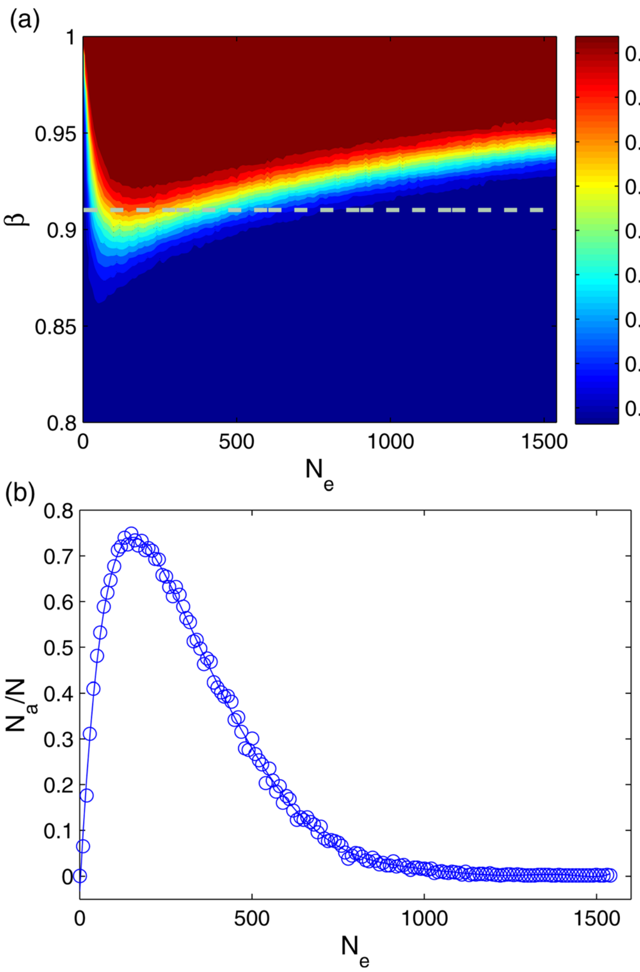

FIG. 2. (Color online) For a small-world network of $N = 1000$ nodes, (a) in the parameter plane (N_e, β), contour plot of the fraction N_a/N of nodes affected by a perturbation to cause the node with the largest degree to overload, and (b) N_a/N versus N_e for fixed photon-generation rate $\beta = 0.91$. Each data point is the average of 1000 network realizations.

this case, a small perturbation that triggered the soft cascading process can lead to a highly heterogeneous weight distribution on the network. The total load of the system becomes $S' = \beta N(N-1)(D'+1)$, where D' is the average length of the shortest paths associated with the new weights. Note that the overall bandwidth is given by $B = \sum B_i = N(N-1)(D+1)$ (since $C = 1$), where D is the average length of the shortest paths associated with the initially homogeneous weight distribution. Typically, we have $D' \geq D$, as a result of the heterogeneous weight distribution. If β approaches unity, a small perturbation can result in a larger value of D', rendering possible the inequality $\beta\,(D'+1) \geq (D+1)$, or $S' \geq B$. The consequence is that the cascading process will not terminate, leading to a constantly changing, *extremely dynamic* network. In particular, the node weights can continue to increase, but no matter how the network adjusts the weights adaptively, jammings cannot be eliminated. That is, under such a condition, the system is unable to reach a jamming-free stable state, for which the weight distribution is homogeneous (although the final node weight is larger than the initial weight). This example illustrates that, when the network operates near its bandwidth limit, a disastrous state can arise where massive, constant jammings occur on the network. In this case, the self-adaptive strategy is not effective to mitigate jamming, and an effective method is to make the weight distribution as homogeneous as possible. This way, although the perturbed node can lose some photons, the overall loss of photons in the whole system can be minimized.

The above analysis is based on considering changes in the photon-generation rate β. To address the effect of network topology on the photon loss, we fix β at some intermediate value where cascades are possible but not persistent and examine the dependence of the ratio N_a/N on N_e, the number of shortcuts. A representative result is shown in Fig. 2(b). We observe the counterintuitive phenomenon that N_a can be maximized by a particular value of N_e. Near this value of N_e, the small-world network is maximally susceptible to photon loss. Equivalently, Fig. 2(b) suggests that, for a small-world type of QKD network, in order to prevent significant photon loss, the number of shortcuts should either be near zero or of the same order of magnitude as the number of nodes in the network.

To gain insights into the dynamical mechanism for the phenomenon in Fig. 2, we can consider some extreme cases where the network has almost no or a large number of shortcuts. To be concrete, we focus on the following three cases where (a) the network has only one shortcut, (b) the network is fully connected so that the number of shortcuts is of the order of N^2, and (c) the number of shortcuts is $0.1 \times N$. For case (a), although the weight of a node connected to the shortcut increases, the length of the shortest paths passing through the node increases only by one. Most node pairs having their shortest paths passing through this node will find that their respective original shortest paths are still the shortest paths as the distribution of the lengths of the shortest paths has been changed little. Redistribution of load occurs only at a few nodes, leading to extremely small value of N_a and nearly zero photon loss. For case (b), the network is fully connected so that the length of all shortest paths is 1. It means that, although the weight of a randomly selected node is increased by 1, it does not affect the shortest paths, and the load of the nodes remains unchanged. In this case, N_a is zero. For case (c), a large number of shortest paths may exist for a given pairs of nodes. In this case, as soon as the weight of the node with the largest bandwidth is increased by 1, many of the shortest paths will be diverted to avoid this node. This can trigger a cascading process and lead to significant photon loss.

The above numerical simulation and physical arguments explain the resonance phenomena in the small-world type QKD networks. We then ask whether the resonant photon loss due to soft cascading can also occur on scale-free and complex clustered networks. We have studied these network structures and found qualitatively similar results to those from the small-world network, as shown in Fig. 3. In particular, for a scale-free QKD network, the average degree should be sufficiently large to reduce the probability of photon loss. For example, for such a network of $N = 1000$ nodes and a photon generating rate $\beta = 0.855$, the average degree should be larger than 10 (Fig. 3(b)). While for a clustered network of 10 clusters, each having 100 nodes and a photon

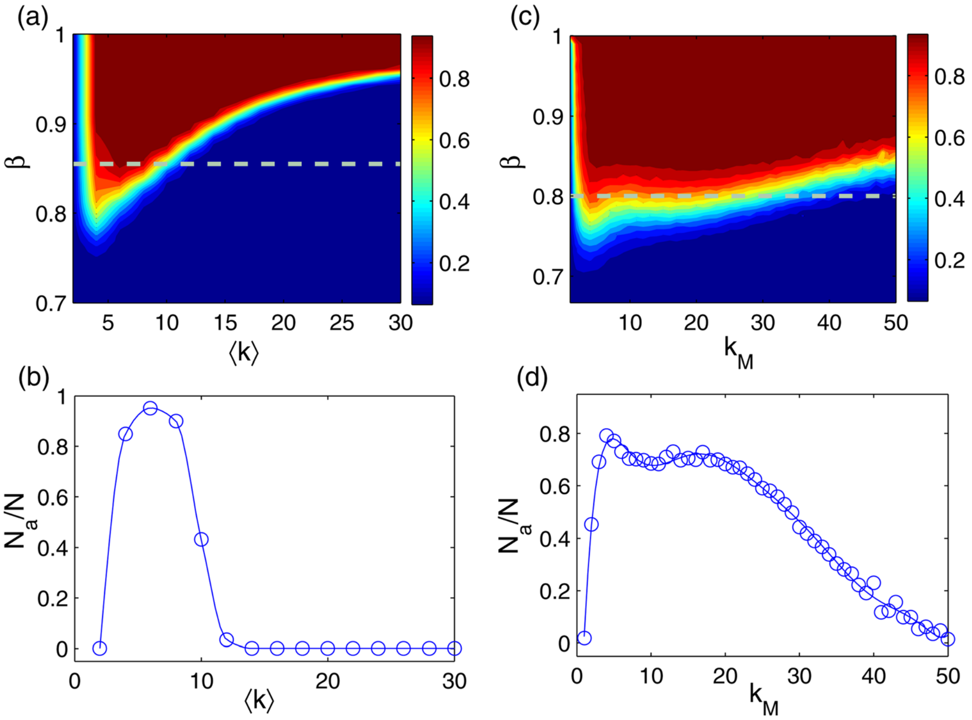

FIG. 3. (Color online) (a) The contour plot of the fraction of affected node N_a/N by perturbing the node with largest bandwidth versus the average degree for scale-free networks. Network size $N = 1000$. (b) A cross section indicated by the dashed line in (a). (c) The contour plot of the fraction of affected node N_a/N versus the average number of inter-cluster links per cluster for clustered networks. $N = 1000$, the number of clusters is $M = 10$. Each cluster is a scale-free subnetwork with average degree 10. (d) A cross section indicated by the dashed line in (c). The photon generating rate β is 0.855 for scale-free networks (b) and 0.8 for clustered networks (d). Each data point is the average of 1000 ensembles.

generating rate $\beta = 0.8$, the number of inter-cluster links per cluster should be at least 50 to prevent photon loss (Fig. 3(d)). Note that when the linkage is sparse, the photon loss is also suppressed. This is because there are not many other routes that photons can be diverted. Although from economical point of view, the cost of sparse linkage is low, and it can mitigate the temporal jamming caused by random flow fluctuations, as proposed in this paper, it could be fragile for node or link hard failures or under attack, i.e., the network can be easily disconnected if a small set of nodes or links are removed. Thus, a robust network that can minimize the impacts of both hard failures and soft malfunctions should have sufficiently dense links that are redundant enough to absorb small perturbations.

To conclude, we have proposed a hybrid architecture for large-scale quantum key distribution networks where long-range communication is relayed by secure repeaters and local communication is sustained by fiber-and-switch subnetworks. We have argued that complex-network characteristics such as the small world, the scale-free, and clustered topologies can be highly relevant to quantum networks. In addition, we have explored the issue of photon loss with respect to these network topologies and found that either sparse or dense connections can be effective to suppress the degree of photon loss. This result is general and independent on particular network structures. It can be anticipated that some form of quantum networks will be designed and tested experimentally in the future, and we hope that our results can provide useful insights into the fundamentals of quantum networks that need to be understood before actual design and implementation. Especially, when more detailed technical modeling of how the entangled photons are received and transmitted by repeaters and switches is available, the difference in their operation and the possible scheme of the topological connection should then be considered and implemented into the model to get an accurate quantum key distribution network structure, and by adding links one by one, the minimum set of links can be identified in order to avoid soft cascading. Since this type of soft cascading also occurs in many other technical systems, such as the digital sensor network, computer routing network, etc., our results are also expected to have impacts in the understanding of the jamming and data loss phenomena in these systems.

This work was supported by AFOSR under Grant No. FA9550-10-1-0083. L.H. was also supported by NSF of China under Grant No. 11005053 and No. 11047606.

[1]M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
[2]*The Physics of Quantum Information*, edited by D. Bouwmeester, A. Ekert, and A. Zeilinger (Springer, Berlin, 2000).
[3]*Quantum Information and Computing*, edited by L. Accardi, M. Ohya, and N. Watanabe (World Scientific, Singapore, 2006).
[4]H. J. Kimble, Nature **453**, 1023 (2008).
[5]C. W. Chou, H. De Riedmatten, D. Felinto, S. V. Polyakov, S. J. Van Enk, and H. J. Kimble, Nature **438**, 828 (2005).
[6]T. Chanelière, D. N. Matsukevich, S. D. Jenkns, S.-Y. Lan, T. A. B. Kennedy, and A. Kuzmich, Nature **438**, 833 (2005).
[7]M. D. Eisaman, A. André, F. Massou, M. Fleischhauer, A. S. Zibrov, and M. D. Lukin, Nature **438**, 837 (2005).
[8]C. Elliott and H. Yeh, "DARPA quantum network testbed," Final Technical Report AFRL-IF-RS-TR-2007-180, July 2007.
[9]H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 5932 (1998).
[10]L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature **414**, 413 (2001).
[11]A. S. Parkins, P. Marte, P. Zoller, and H. J. Kimble, Phys. Rev. Lett. **71**, 3095 (1993).
[12]J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997).
[13]S. J. van Enk, J. I. Cirac, and P. Zoller, Science **279**, 205 (1998).

[14]M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[15]D. Matsukevich and A. Kuzmich, Science **306**, 663 (2004).
[16]R. Miller, T. E. Northup, K. M. Birnbaum, A. Boca, A. D. Boozer, and H. J. Kimble, J. Phys. B **38**, S551 (2005).
[17]T. Wilk, S. C. Webster, A. Kuhn, and G. Rempe, Science **317**, 488 (2007).
[18]J. F. Sherson, H. Krauter, R. K. Olsson, B. Julsgaard, K. Hammerer, I. Cirac, and E. S. Polzik, Nature **443**, 557 (2006).
[19]J. Laurat, K. S. Choi, H. Deng, C. W. Chou, and H. J. Kimble, Phys. Rev. Lett. **99**, 180504 (2007).
[20]L. Jiang, J. M. Taylor, and M. D. Lukin, Phys. Rev. A **76**, 012301 (2007).
[21]O. A. Collins, S. D. Jenkins, A. Kuzmich, and T. A. B. Kennedy, Phys. Rev. Lett. **98**, 060502 (2007).
[22]N. Sangouard, C. Simon, B. Zhao, Y.-A. Chen, H. de Riedmatten, J.-W. Pan, and N. Gisin, Phys. Rev. A **77**, 062301 (2008).
[23]Z.-S. Yuan, Y.-A. Chen, B. Zhao, S. Chen, J. Schmiedmayer, and J.-W. Pan, Nature **454**, 1098 (2008).
[24]A. E. Motter and Y.-C. Lai, Phys. Rev. E **66**, 065102(R) (2002).
[25]L. Zhao, K. Park, and Y.-C. Lai, Phys. Rev. E **70**, 035101(R) (2004).
[26]L. Zhao, K. Park, Y.-C. Lai, and N. Ye, Phys. Rev. E **72**, 025104(R) (2005).
[27]L. Huang, L. Yang, and K. Yang, Phys. Rev. E **73**, 036102 (2006).
[28]This assumption is required to maximize the throughput of the quantum network. Although this may introduce vulnerability in the sense of traffic analysis (Ref. 8), it can be mitigated by a random weight perturbation.
[29]D. J. Watts and S. H. Strogatz, Nature **393**, 440 (1998).
[30]M. E. J. Newman and D. J. Watts, Phys. Rev. E **60**, 7332 (1999).
[31]K. Yang, L. Huang, and L. Yang, Phys. Rev. E **70**, 015102(R) (2004).
[32]A.-L. Barabási and R. Albert, Science **286**, 509 (1999).